Information Sciences and Technological Innovations

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Research Expansion Alliance

Inf. Sci. Technol. Innov. Vol. 2, No. 3 (2025) 217–225.

Paper Type: Original Article

PM4Sand-Based Nonlinear Constitutive Modeling of Sand Behavior under Seismic Loading

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Citation:

Received: 19 February 2025 *Revised: 25 April 2025* *Accepted: 27 July 2025*	Azimi, S. E. (2025). PM4Sand-based nonlinear constitutive modeling of sand behavior under seismic loading. *Information sciences and technological innovations,* 2(3), 217- 225.

Abstract

This paper presents an analytical review of nonlinear constitutive modeling of granular soils under seismic loading with emphasis on the PM4Sand model. The revised manuscript addresses a central problem in the original draft: the earlier version was written with the rhetoric of an original numerical study, while it did not report a reproducible simulation program, calibrated datasets, or quantitative results. In the present version, the study is explicitly reframed as a structured literature-based review and engineering workflow. The paper first clarifies the mechanics of nonlinear sand response, including stiffness degradation, hysteretic damping, excess pore-pressure generation, cyclic mobility, and post-liquefaction deformation. It then synthesizes how PM4Sand has been implemented, calibrated, validated, and used in recent earthquake-geotechnical applications. A practical workflow is proposed for model selection, calibration evidence, response metrics, and engineering interpretation. The review further consolidates reported response trends with respect to seismic intensity, density state, and confinement, and critically discusses the benefits and limitations of PM4Sand in site response, liquefaction assessment, and soil-structure interaction problems. The main novelty of the paper lies in integrating constitutive theory, calibration logic, and engineering interpretation into a single analytical framework that can support future simulation-based studies and practice-oriented applications. This study is positioned as an analytical and workflow-oriented contribution, aiming to synthesize existing evidence and provide a structured framework for future simulation-based research.

Keywords: PM4S, Nonlinear soil behavior, Seismic loading, Liquefaction, Constitutive modeling, Analytical review, Earthquake geotechnics.

1 | Introduction

The nonlinear response of soils during earthquakes is one of the defining issues of modern geotechnical earthquake engineering because the most damaging consequences of strong shaking often arise from the interaction between seismic demand and the evolving stiffness, strength, drainage condition, and effective

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https://doi.org/10.48314/isti.vi.48

stress state of the ground. In granular deposits, seismic loading may trigger shear modulus degradation, hysteretic energy dissipation, excess pore-pressure generation, cyclic mobility, liquefaction, and post-liquefaction deformation. These mechanisms are strongly path-dependent and cannot be represented adequately by purely linear or oversimplified constitutive assumptions. Recent studies on one-dimensional and multidimensional site response, reclaimed soils, and liquefiable deposits have repeatedly shown that the computed response becomes highly sensitive to constitutive selection once the loading level moves beyond the small-strain range [1–4].

In practical earthquake engineering, inadequate constitutive representation affects liquefaction assessment, embankment and retaining-wall deformation analysis, settlement estimation, and soil-structure interaction. If stiffness loss is underestimated, the computed response may remain unrealistically elastic. If pore-pressure buildup or post-triggering deformation is exaggerated or misrepresented, the predicted deformation demand may become either unconservative or excessively conservative. For this reason, constitutive models that can reproduce both triggering and deformation have become increasingly important in performance-based design and hazard-informed assessment frameworks [3], [4].

Among effective-stress constitutive models, PM4Sand has become one of the most influential frameworks for seismic analyses of sands and nonplastic granular soils. It was developed specifically for earthquake engineering applications and has been implemented, verified, and used in numerical platforms such as Fast Lagrangian Analysis of Continua (FLAC) and Open System for Earthquake Engineering Simulation (OpenSees). Its appeal lies in its ability to represent stress-path dependence, pore-pressure generation, cyclic mobility, contraction-dilation transition, and post-liquefaction response within a practical constitutive structure. The model has also been applied beyond ideal clean sands, including mine tailings, gravelly materials, liquefiable embankment foundations, layered liquefiable deposits, and retaining structures [5–10].

A major weakness of the original draft was the mismatch between the article type and the evidence presented. The manuscript used the language of an original research paper, but it did not report an actual calibrated simulation program, a reproducible dataset, or quantitative outputs. Accordingly, the paper is more defensible and more publishable in its present revised form as an analytical review and engineering workflow paper. This repositioning resolves the earlier over-claiming problem and aligns the title, abstract, section logic, and conclusions with the available evidence.

The research gap addressed here is therefore not a missing constitutive model, but a missing integrative workflow in the literature. Existing PM4Sand studies often focus on one implementation, one case history, one calibration exercise, or one application domain. What is still less common is a single paper that connects motivation, constitutive theory, calibration logic, response metrics, engineering interpretation, and limitations into one coherent article that can be used by both researchers and practitioners.

The novelty of this paper is analytical rather than experimental. Specifically, the paper contributes:

I. An explicit reframing of PM4Sand use from an isolated model application to a full engineering workflow.

II. A structured synthesis of the evidence needed for defensible calibration, including laboratory, field, and boundary-value checks.

III. A unified interpretation of reported PM4Sand response trends with respect to seismic intensity, relative density, and effective confinement, and

IV. A critical statement of what PM4Sand can support reliably in practice and where caution is still required.

Accordingly, the objective of this article is to develop a structured and technically grounded review of nonlinear constitutive modeling of sand behavior under seismic loading using PM4Sand, while clearly separating evidence synthesis from original numerical validation. The remainder of the paper is organized as follows. Section 2 reviews the constitutive basis of nonlinear sand response and PM4Sand. Section 3 develops an evidence-based calibration and application workflow. Section 4 synthesizes reported nonlinear response trends and engineering implications. Section 5 discusses limitations and future research needs. Section 6 concludes the paper.

2 | Theoretical Background of Nonlinear Soil Behavior and PM4S

2.1 | Nonlinear Response of Sands under Seismic Loading

The seismic response of granular soils is inherently nonlinear because the stress–strain relation evolves continuously during cyclic loading. At very small strains, the response is approximately elastic and may be represented by the small-strain shear modulus:

$$\mathrm{Gmax} = \rho\ \mathrm{Vs}^2, \tag{1}$$

where ρ is the mass density, and Vs is the shear-wave velocity.

As cyclic strain increases, the secant shear modulus decreases, and the material dissipates energy through hysteresis. The corresponding damping ratio may be expressed as:

$$\xi = \Delta \mathrm{W} / (4\pi\ \mathrm{Ws}), \tag{2}$$

where ΔW is the energy dissipated in one cycle, and Ws is the maximum stored strain energy.

For saturated sands, cyclic shearing alters effective stress through excess pore-pressure generation. A convenient response index is the excess pore-pressure ratio:

$$\mathrm{ru} = \Delta \mathrm{u} / \sigma'\mathrm{v0}, \tag{3}$$

where Δu is the excess pore-water pressure, and $\sigma'v0$ is the initial vertical effective stress. As ru approaches unity, the available effective stress is reduced sharply, and the soil may enter cyclic mobility or liquefaction [5], [10–12]. These basic relations clarify why earthquake-induced soil behavior cannot be assessed solely from peak acceleration or linear amplification factors. The same input motion can produce markedly different outcomes depending on density state, overburden stress, drainage conditions, stress path, and constitutive memory. Studies of liquefiable deposits and site-specific nonlinear response show that important engineering outcomes depend on the coupled evolution of stiffness degradation, damping, pore-pressure generation, and strain concentration rather than on any one variable in isolation [2], [10–14]. Effective-stress constitutive modeling is therefore not a cosmetic refinement. For problems involving liquefaction-prone sands, seismic settlements, embankments, or soil-structure systems, it is often the only internally consistent way to represent the governing mechanics. Equivalent-linear procedures remain useful for preliminary evaluation, but they cannot capture the progressive coupling among stiffness degradation, hysteretic damping, pore-pressure accumulation, and post-liquefaction deformation [1–4], [10], [15–17].

2.2 | PM4Sand as a Constitutive Framework for Seismic Geotechnics

PM4Sand was developed as a practical, effective stress plasticity model for sands in earthquake engineering applications. It is based on a stress-ratio-controlled, critical-state-compatible, bounding-surface plasticity framework that was adapted specifically to improve engineering usability for cyclic loading, liquefaction triggering, and post-triggering deformation assessment. The model is attractive because it balances constitutive sophistication with calibration practicality. It is advanced enough to capture state dependence, pore-pressure generation, contraction-dilation behavior, and cyclic mobility, but structured enough to be calibrated from laboratory and field information that is meaningful in practice [5], [6].

The PM4Sand literature also shows that implementation quality matters. Verification studies in OpenSees and other numerical environments demonstrate that reliable numerical integration and model deployment are prerequisites for trustworthy large-scale simulations. At the same time, later studies extend the discussion from implementation to calibration against mine tailings, cyclic gravel response, layered liquefiable deposits, embankment foundations, and soil-structure interaction. These developments are important because they show that PM4Sand should not be evaluated only at the single-element level; its engineering value depends on how well element-level calibration transfers to boundary-value response [6–10], [18–23].

A central lesson in the literature is that PM4Sand allows nonlinear behavior to emerge from constitutive evolution rather than from externally imposed modulus-reduction and damping curves alone. This constitutive framework gives the analyst a more physically meaningful basis for interpreting the transition from nearly elastic response to strong nonlinearity, and from pre-liquefaction cyclic loading to post-liquefaction deformation. However, the model remains calibration-sensitive. If contraction tendency, initial stiffness, state condition, or post-triggering deformation characteristics are misrepresented, the resulting analysis may still be numerically stable but physically misleading [7–9], [13, 14], [18–23].

3 | Analytical Calibration and Application Workflow

Because this paper does not present a new simulation dataset, Section 3 is intentionally written as an evidence-based workflow rather than as a methods section for an original numerical experiment. This change resolves one of the most serious weaknesses of the earlier draft. The goal here is to synthesize what a defensible PM4Sand study should contain on the basis of recent literature, and to organize that evidence in a way that is directly useful for future simulation-based work and scientific reporting.

3.1 | Calibration Logic Derived from the Literature

A rigorous PM4Sand study begins with calibration, not simulation. The calibration stage should constrain small-strain stiffness, cyclic resistance, pore-pressure generation rate, dilation-contraction tendency, and post-liquefaction deformation behavior by using laboratory and field evidence together. Recent studies on tailings, gravelly materials, post-liquefaction settlement, and liquefiable boundary-value problems converge on the same conclusion: calibration should be multi-objective and response-based, not dependent on a single target such as triggering cycle count alone [7–9], [13], [14], [16], [24].

In engineering terms, calibration links measurable inputs to target constitutive behavior. Grain characteristics, density indices, and Cone Penetration Test (CPT) / Standard Penetration Test (SPT) information help define the soil state. Shear-wave velocity and resonant-column data constrain the initial stiffness level. Cyclic Direct Simple Shear (DSS) or triaxial tests inform cyclic resistance and pore-pressure buildup tendency. Field manifestation evidence, such as settlement, ejecta severity, or deformation patterns, provides a boundary-value check that prevents calibration from becoming detached from actual site behavior [5], [7], [11–14], [16], [24]. *Table 1* presents the calibration framework synthesized from recent PM4Sand-related studies.

Table 1. Calibration framework synthesized from recent PM4Sand-related studies.

Calibration Component	Typical Evidence Source	Main Target Response	Why it Matters
Initial stiffness	Resonant column, field geophysics	Gmax and strain threshold for nonlinearity	Controls impedance contrast and onset of modulus reduction
Density/state condition	Relative density, CPT/SPT trends, index tests	Cyclic resistance level and dilation tendency	Governs susceptibility to liquefaction and cyclic mobility
Cyclic resistance	Cyclic DSS or cyclic triaxial tests	Triggering resistance and pore-pressure buildup	Needed to represent excess pore-pressure generation realistically
Stress-path dependence	Monotonic and cyclic element tests	Hysteretic response under varying confinement and bias	Important for realistic loading path representation
Post-triggering deformation	Centrifuge tests, shake-table data, and settlement records	Residual strain and reconsolidation response	Required for deformation-focused design

Field manifestation check	Ejecta, settlement, embankment, or wall response	Boundary-value realism	Keeps calibration engineering consistent

A compact way to formalize the calibration target is through a response vector:

$$R = [G/Gmax,\ \xi,\ ru,\ \gamma max,\ \varepsilon v{,}post]^T, \tag{4}$$

where G/Gmax is the normalized secant modulus, ξ is the damping ratio, ru is the excess pore-pressure ratio, γmax is the maximum cyclic shear strain, and εv, post is a post-liquefaction volumetric strain or settlement-related deformation index. In a defensible PM4Sand study, calibration should be assessed by how consistently the model reproduces this response vector across the expected range of loading conditions.

3.2 | Recommended Nonlinear Analysis Matrix for Future PM4S and Studies

Once calibrated, the model should be evaluated in a parametric seismic analysis program that examines the effects of seismic intensity, density state, and confinement. A single representative simulation is rarely sufficient. The most useful conclusions typically emerge from trends rather than from one isolated case. This point is retained from the original manuscript, but it is now presented correctly as methodological guidance drawn from the literature rather than as a reported analysis program from the present paper [3], [4], [10], [15–24]. *Table 2* presents the recommended parametric nonlinear analysis matrix for future PM4Sand studies.

Table 2. Recommended parametric nonlinear analysis matrix for future PM4Sand studies.

Variable	Level 1	Level 2	Level 3	Typical Outputs
Seismic intensity	Low	Moderate	High	Amplification, γmax, ru, settlement tendency
Relative density	Loose	Medium-dense	Dense	Triggering resistance, cyclic mobility, and residual deformation
Effective confinement	Low	Medium	High	Stiffness degradation, damping evolution, pore-pressure growth
Analysis scale	Element test	1D profile	2D boundary-value problem	Calibration consistency and transferability
Post-earthquake state	Immediate	Short-term dissipation	Reconsolidated state	Settlement and residual response

4 | Critical Synthesis of Reported PM4S and Responses

Because no new numerical results are reported in this paper, the discussion below is framed explicitly as a synthesis of published PM4Sand-related studies. This editorial change eliminates the earlier ambiguity between original findings and literature-based interpretation.

4.1 | Reported Nonlinear Response Trends

Published PM4Sand-based studies consistently indicate a progression from near-linear to strongly nonlinear behavior as seismic intensity increases. Under low shaking levels, hysteresis loops remain narrow, secant stiffness stays close to the small-strain regime, and excess pore-pressure generation is limited. As input intensity increases, the response becomes dominated by stiffness degradation, broader hysteresis, increasing damping, and faster pore-pressure accumulation. These constitutive shifts can modify both motion characteristics and deformation indicators, including spectral amplification, internal strain demand, and residual settlement tendency [3], [4], [10], [15], [17].

A second repeated trend concerns the density state. Loose sands generally show the most rapid transition into strong nonlinearity because their contractive tendency promotes pore-pressure accumulation and reduces the effective stress available to maintain stiffness. In contrast, dense sands usually retain stiffness longer, develop smaller permanent strains, and exhibit more controlled cyclic response at the same shaking level. Medium-dense soils occupy an intermediate position. These relationships are reported across studies involving layered deposits, embankments, retaining walls, reclaimed soils, and other boundary-value problems [8], [9], [15], [16], [18–24].

A third important trend concerns confinement. Increasing effective confinement commonly delays strong stiffness degradation and reduces the rate of pore-pressure buildup for the same cyclic demand. In layered profiles, this often leads to vertical differentiation of response, where shallow low-confinement layers accumulate larger cyclic strains and higher ru values, while deeper layers contribute more strongly to impedance control and strain filtering. This vertical differentiation is an important engineering insight because it implies that critical nonlinear behavior may localize within specific zones rather than distribute uniformly through the soil column [10], [15–21], [23, 24].

Finally, the literature suggests that PM4Sand is especially valuable when the engineering question concerns deformation rather than triggering alone. Triggering procedures can often indicate whether a layer is liquefiable, but they do not explain how much residual strain, settlement, or wall movement will occur once stiffness and effective stress evolve during shaking. Studies on sheet-pile walls, embankments, ejecta severity, reconsolidation settlement, and reclaimed soils repeatedly highlight the importance of constitutive treatment of post-triggering mechanics [11–14], [16], [18–23]. *Table 3* presents the synthesized PM4Sand response trends reported across recent studies.

Table 3. Synthesized PM4Sand response trends reported across recent studies.

Increasing Factor	Shear Modulus Retention	Hysteretic Damping	Excess Pore-Pressure Ratio	Engineering Interpretation
Seismic intensity	Decreases	Increases	Increases	Stronger mobilization of nonlinear and liquefaction-related mechanisms
Relative density	Increases	Slight to moderate reduction in severe cases	Decreases	Denser soils are more resistant to cyclic mobility
Effective confinement	Increases	Moderate change	Slower buildup	Higher confinement generally stabilizes the response
Loose shallow layers	Strong decrease	Strong increase	Rapid increase	Most critical for triggering and deformation
Dense deeper layers	Moderate decrease	Moderate increase	Slower increase	More important for impedance and strain filtering

4.2 | Engineering Implications, Strengths, and Cautions

From a design perspective, the principal advantage of PM4Sand is that it provides a constitutive route from material state to engineering demand. Instead of prescribing a fixed modulus-reduction relationship disconnected from pore-pressure evolution, the analyst obtains a response that links stiffness degradation, damping, liquefaction, and deformation within one effective-stress framework. This unified effective-stress framework is particularly important in site response, embankment evaluation, retaining-wall response, and nonlinear soil-structure interaction [6], [10–16], [18–23]. A second implication is methodological. The literature makes clear that PM4Sand should not be used with unexamined default parameters when design or hazard decisions depend on deformation prediction. A professional PM4Sand study should report calibration evidence, target responses, sensitivity checks, the chosen analysis dimensionality, and the rationale behind major assumptions. Without such transparency, even an advanced effective-stress analysis may project a misleading sense of reliability [5–9], [13], [14], [23], [24]. At the same time, PM4Sand has practical limitations. Predictive quality depends strongly on calibration discipline, especially for post-triggering deformation. Uncertainty in stratigraphy, drainage, spatial variability, and boundary representation can affect results as much as constitutive parameter choice. One-dimensional analyses, while useful and efficient, may under-represent localized deformations and some soil-structure interaction effects. Moreover, not all granular geomaterials behave like the clean sands for which PM4Sand was originally developed; extension to mine tailings, gravelly soils, or unusual fills requires careful validation [7], [9], [15–24].

5 | Conclusion

This paper has presented a revised and internally consistent analytical review of nonlinear constitutive modeling of sand behavior under seismic loading using PM4Sand. The central revision was to align the manuscript type with the available evidence. Rather than claiming a new simulation study without a reproducible numerical program, the paper now explicitly offers an analytical synthesis of constitutive theory, calibration logic, reported nonlinear response trends, and engineering interpretation. The main scientific value of the revised manuscript lies in integrating PM4Sand motivation, theory, calibration evidence, and application logic into a single workflow-oriented paper. The review shows that PM4Sand is most useful when the engineering question extends beyond liquefaction triggering to deformation, residual response, and soil-structure interaction. At the same time, it emphasizes that the model is only as defensible as the calibration strategy, response metrics, and reporting transparency that support it. In its current revised form, the paper is substantially better suited to a scientific-review or analytical-methodology journal format than the original draft. It can also serve as a strong conceptual foundation for a later simulation-based article built on actual PM4Sand calibration and reproducible numerical analyses.

The present paper resolves many editorial and conceptual weaknesses of the original draft, but it does not solve every limitation that would matter for a high-tier original research article. Most importantly, this paper still does not present new numerical simulations, laboratory data, calibration files, or quantitative validation results. Accordingly, it should be judged as an analytical review article rather than as an original simulation paper. The paper also remains dependent on the quality and coverage of the cited literature. Although the synthesis is structured and technically grounded, it cannot replace direct model verification, sensitivity analysis, or benchmark validation. In other words, the present article can guide a future PM4Sand study, but it cannot substitute for one. Future work should move in three directions. First, future manuscripts should combine PM4Sand calibration with field-verified case histories so that the model can be evaluated at both element and system scales. Second, the effects of spatial variability, partial drainage, and multidirectional loading should be explored more systematically for layered, reclaimed, and infrastructure-adjacent sites. Third, additional work is needed on calibration transferability to gravelly soils, mine tailings, and engineered fills.

Acknowledgments

The authors would like to thank the editors and reviewers for their valuable comments.

Author Contribution

S. E. A.: conceptualization, methodology, validation, formal analysis, investigation, writing-creating the initial design, writing-reviewing and editing, visualization.

Funding

This research received no funding.

Data Availability

The data will be available on request from the corresponding author.

Conflicts of Interest

The authors declare no conflict of interest.

Consent for Publication

The author confirms consent for the publication of this work

Ethics Approval and Consent to Participate

This article does not contain any studies with human participants performed by the author.

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