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Note for Biochemical/Electrochemical/Physicochemical Hypergraphs and Superhypergraphs

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Abstract

A *Chemical Graph* represents a molecule where atoms are vertices and chemical bonds are edges, thereby modeling molecular structure mathematically (cf.[1, 2, 3, 4, 5]). A *Chemical Hypergraph* is a specialized multilevel hypergraph that models an entire chemical system by representing atoms, chemical bonds, molecules, and reactions as layered hyperedges across different levels (cf.[6, 7, 8]). A *Chemical Superhypergraph* is a hierarchical, multi-level structure that models atoms, chemical bonds, molecular substructures, complete molecules, and higher-order aggregates as nested hyperedges, each associated with quantitative weights.

In this paper, we investigate whether new concepts such as the *Biochemical Graph*, *Electrochemical Graph*, *Physicochemical Graph*, and *Medichemical Graph* can be formally defined. We also explore whether their corresponding *HyperGraph* and *SuperHyperGraph* extensions can be constructed. This work is primarily a theoretical study conducted at a conceptual level; however, we expect that future research by domain experts will further examine the practical effectiveness and applications of these proposed frameworks.

Keywords: Superhypergraphs, Hypergraphs, Biochemical graph, Electrochemical graph, Physicochemical graph, Chemical graph.

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1 | Preliminaries

This section presents the fundamental concepts and definitions that underpin the discussions in this paper. Unless otherwise noted, all graphs considered here are *finite*.

1.1 | SuperHyperGraphs

A finite *hypergraph* generalizes the classical graph model by permitting *hyperedges* that connect any non-empty subset of vertices [9, 10, 11]. Building on this concept, a finite *SuperHyperGraph* is obtained by iteratively applying the powerset operator, thereby creating nested hierarchies of vertex and edge sets that encode multi layered relationships [12, 13]. Such structures have demonstrated utility in areas ranging from molecular design and complex-network analysis to advanced signal-processing pipelines [14, 15, 16]. Unless stated otherwise, the integer n in $P_n(\cdot)$ or in an n-SuperHyperGraph is assumed to be non-negative.

Definition 1 (Base Set)**.** A *base set* S is the initial universe of discourse:

$$S \;=\; \{\, x \mid x \text{ belongs to the context at hand}\}.$$

Every element that appears in $\mathcal{P}(S)$ or in any iterated powerset $\mathcal{P}_n(S)$ must of course lie in S.

Definition 2 (Powerset)**.** (cf.[17, 18]) For a set S, the *powerset* $\mathcal{P}(S)$ is the family of all subsets of S:

$$\mathcal{P}(S) \;=\; \{\, A \subseteq S \,\}.$$

This collection includes both S itself and the empty set $\emptyset$.

Definition 3 (Hypergraph)**.** [19, 20] A *hypergraph* is an ordered pair $H = (V, E)$ where

- V is a finite vertex set, and
- E is a finite family of non-empty subsets of V; the members of E are called *hyperedges*.

Hypergraphs naturally represent interactions that involve more than two participants.

Example 1 (Real-life hypergraph: study groups in a library)**.** Let $V = \{\text{Alice}, \text{Bob}, \text{Cara}, \text{Dan}, \text{Eva}\}$ be students who meet to study. Define hyperedges as multi-person study sessions:

$$E = \big\{\{\text{Alice}, \text{Bob}, \text{Cara}\},\ \{\text{Bob}, \text{Dan}\},\ \{\text{Alice}, \text{Dan}, \text{Eva}\},\ \{\text{Cara}, \text{Eva}\}\big\}.$$

Then $H = (V, E)$ is a hypergraph (Definition 1.3): each $e \in E$ records one session and can involve two or more participants, naturally capturing group interactions.

Definition 4 (n-th Powerset)**.** [21, 22] Let X be a set. The first powerset is $\mathcal{P}_1(X) = \mathcal{P}(X)$. For $n \geq 1$ we define

$$\mathcal{P}_{n+1}(X) \;=\; \mathcal{P}\big(P_n(X)\big).$$

When the empty set is excluded one writes $\mathcal{P}^*_n(X) = \mathcal{P}_n(X) \setminus \{\emptyset\}$.

Example 2 (Real-life n-th powerset: access bundles and policies)**.** Consider a workplace with two basic permissions $X = \{\text{Email}, \text{Calendar}\}$. The first powerset lists all permission bundles:

$$\mathcal{P}_1(X) = \mathcal{P}(X) = \big\{\, \varnothing,\ \{\text{Email}\},\ \{\text{Calendar}\},\ \{\text{Email}, \text{Calendar}\} \,\big\}.$$

Elements of the second powerset $\mathcal{P}_2(X) = \mathcal{P}(\mathcal{P}_1(X))$ are *policies*, i.e., sets of bundles. For instance,

$$\varnothing, \quad \{\varnothing\}, \quad \{\{\text{Email}\}, \{\text{Calendar}\}\},$$
$$\{\{\text{Email}, \text{Calendar}\}\}, \quad \mathcal{P}_1(X) \text{ (the policy allowing any bundle).}$$

In practice: $\{\{\text{Email}\}\}$ means "only the Email bundle is an allowed configuration," while $\mathcal{P}_1(X)$ means "any bundle is allowed." This concretely illustrates Definition 1.5.

Definition 5 (n-SuperHyperGraph)**.** (cf. [23]) Fix a finite, nonempty base set V_0 and define the iterated powerset by

$$\mathcal{P}^0(V_0) \coloneqq V_0, \qquad \mathcal{P}^{k+1}(V_0) \coloneqq \mathcal{P}\big(\mathcal{P}^k(V_0)\big) \quad (k \in \mathbb{N}).$$

For an integer $n \geq 0$, an *n-SuperHyperGraph on V_0* is a pair

$$\text{SHG}^{(n)} = (V, E)$$

such that

$$V \subseteq \mathcal{P}^n(V_0) \quad \text{and} \quad E \subseteq \mathcal{P}(V) \setminus \{\varnothing\}.$$

Elements of V are called *n-supervertices* and elements of E are *n-superedges*. (In particular, each n-superedge is a nonempty subset of V.)

Example 3 (Real-life n-SuperHyperGraph: programs made of teams (two levels up))**.** Let the base set be individual engineers

$$V_0 = \{\text{A}, \text{B}, \text{C}, \text{D}\}.$$

Then $\mathcal{P}^1(V_0) = \mathcal{P}(V_0)$ are *teams* (subsets of engineers) and $\mathcal{P}^2(V_0) = \mathcal{P}(\mathcal{P}(V_0))$ are *programs* (sets of teams). Take $n = 2$ and choose the following 2-supervertices (programs)

$$V = \big\{v_1 = \{\{\text{A}, \text{B}\}, \{\text{C}\}\},\ v_2 = \{\{\text{B}, \text{C}\}, \{\text{D}\}\},\ v_3 = \{\{\text{A}, \text{D}\}\}\big\} \ \subseteq\ \mathcal{P}^2(V_0).$$

Define 2-superedges (joint portfolios) as nonempty subsets of V:

$$E = \big\{\ \{v_1, v_2\},\ \{v_2, v_3\}\ \big\} \ \subseteq\ \mathcal{P}(V) \setminus \{\varnothing\}.$$

Then $\text{SHG}^{(2)} = (V, E)$ satisfies Definition 1.7. Interpretation: v_1 is a program comprising teams $\{\text{A}, \text{B}\}$ and $\{\text{C}\}$; the superedge $\{v_1, v_2\}$ models a portfolio that links two programs which share resources or milestones.

Definition 6 (Flattening at level n)**.** Let E_0 be a finite base set (*atoms*). For $n \in \mathbb{N}_0$ and $A \in \mathcal{P}^n(E_0)$, define the *flattening* map

$$\text{Flat}_0(e) := \{e\} \quad (e \in E_0), \qquad \text{Flat}_{n+1}(A) := \bigcup_{B \in A} \text{Flat}_n(B).$$

Thus $\text{Flat}_n(A) \subseteq E_0$ is the set of all base atoms that occur inside A. For $s \in E_0$ and $A \in \mathcal{P}^n(E_0)$, set the indicator

$$\iota_n(A, s) := \begin{cases} 1, & s \in \text{Flat}_n(A), \\ 0, & \text{otherwise.} \end{cases}$$

1.2 | Chemical Graph, HyperGraph, and SuperHyperGraph

A Chemical Graph represents a molecule where atoms are vertices and chemical bonds are edges, modeling molecular structure mathematically (cf.[1, 2, 3, 4, 5]). A Chemical Hypergraph is a specialized multilevel hypergraph that models an entire chemical system by representing atoms, chemical bonds, molecules, and reactions as layered hyperedges across different levels (cf.[6, 7, 8]). A Chemical Superhypergraph is a hierarchical, multi-level structure that models atoms, chemical bonds, molecular substructures, complete molecules, and higher-order aggregates as nested hyperedges, each associated with quantitative weights.

Definition 7 (Chemical Graph)**.** A *chemical graph* is a finite labeled graph

$$G = (V, E, \lambda_V, \lambda_E),$$

where (V, E) is a simple undirected graph, $\lambda_V : V \to \Sigma_V$ assigns to each vertex (atom) its label (e.g. element, optional isotope and formal charge), and $\lambda_E : E \to \Sigma_E$ assigns to each edge (bond) its label (e.g. bond order in

$\{1, 2, 3, \text{arom}\}$ and optional stereochemical tag). Each connected component of G corresponds to one molecular component (e.g. salt ion or solvent).

A *hydrogen-suppressed* chemical graph is a pair (G, h) with $G = (V, E, \lambda_V, \lambda_E)$ as above and $h : V \to \mathbb{N}_0$ giving the number of implicit hydrogens at each vertex. Its hydrogen-expanded form $\widehat{G}$ is the labeled graph with

$$\widehat{V} = V \ \cup \ \{(v, i) : v \in V, \ 1 \le i \le h(v)\}, \qquad \widehat{E} = E \ \cup \ \big\{\{v, (v, i)\} : v \in V, \ 1 \le i \le h(v)\big\},$$

where each (v, i) is labeled as a hydrogen atom. Two hydrogen-suppressed graphs are *chemically equivalent* if their expanded graphs are isomorphic as labeled graphs.

Example 4 (Chemical Graph: Ethanol (hydrogen–suppressed form))**.** Let the hydrogen–suppressed molecular graph of ethanol be

$$G = (V, E, \lambda_V, \lambda_E), \qquad V = \{C_1, C_2, O_1\}, \quad E = \big\{\{C_1, C_2\}, \{C_2, O_1\}\big\}.$$

Vertex labels encode elements $\lambda_V(C_1) = \lambda_V(C_2) = \mathsf{C}$, $\lambda_V(O_1) = \mathsf{O}$, and edge labels encode bond orders $\lambda_E(\{C_1, C_2\}) = \lambda_E(\{C_2, O_1\}) = 1$. Attach implicit hydrogens by

$$h : V \to \mathbb{N}_0, \qquad h(C_1) = 3, \ \ h(C_2) = 2, \ \ h(O_1) = 1,$$

so the hydrogen–expanded graph $\widehat{G}$ has three H atoms on C_1, two on C_2, and one on O_1, i.e. C_2H_6O.

Definition 8 (Multilevel Hypergraph)**.** A *multilevel hypergraph* of depth n is a tuple

$$\mathcal{H}^{(n)} \ = \ \big(V_0, \, E_1, E_2, \ldots, E_n, \, w_1, w_2, \ldots, w_n\big),$$

where:

- V_0 is a finite set of *base vertices*.
- $E_1 \subseteq \mathcal{P}(V_0) \setminus \{\emptyset\}$ is the set of *first-level hyperedges*, each a nonempty subset of V_0.
- For each $\ell = 2, \ldots, n$,
$$E_\ell \ \subseteq \ \mathcal{P}(E_{\ell-1}) \setminus \{\emptyset\}$$
is the set of *ℓ-th-level hyperedges*, each a nonempty collection of $(\ell - 1)$-level hyperedges.
- $w_\ell \colon E_\ell \to \mathbb{R}_{\ge 0}$ is an optional *weight function* on level-ℓ hyperedges.

In particular:

- Level 1 hyperedges connect base vertices (ordinary hypergraph).
- Level 2 hyperedges connect groups of level 1 hyperedges, capturing a second layer of grouping.
- ...
- Level n hyperedges connect groups of level $(n - 1)$ hyperedges, enabling up to n nested layers of higher-order relationships.

Definition 9 (Chemical Hypergraph)**.** (cf.[6, 7, 8]) A *chemical hypergraph* modeling an entire chemical system is a multilevel hypergraph

$$\mathcal{H} = \big(V_0, \, E_S, \, E_N, \, E_D\big),$$

where:

- V_0 is the finite set of atomic nodes (each $v \in V_0$ is an atom).
- $E_S \subseteq \mathcal{P}(V_0)$ is the set of *simple hyperedges* (chemical bonds or molecular substructures), each $e_S \in E_S$ satisfying $e_S \neq \emptyset$.
- $E_N \subseteq \mathcal{P}(E_S)$ is the set of *nesting hyperedges* (molecules), each $e_N \in E_N$ a nonempty subset of E_S indicating which bonds/substructures form that molecule.
- $E_D \subseteq E_N \times E_N$ is the set of *directed hyperedges* (chemical reactions), each $(e_{N,r} \to e_{N,p}) \in E_D$ indicating that reactant molecule-hyperedge $e_{N,r}$ yields product molecule-hyperedge $e_{N,p}$.

Thus, a chemical hypergraph represents atoms by nodes, bonds/substructures by simple hyperedges, molecules by nesting hyperedges, and reactions by directed hyperedges, all within a single uniform multilevel structure.

Example 5 (Chemical Hypergraph: Neutralization $\mathrm{HCl} + \mathrm{NaOH} \to \mathrm{NaCl} + \mathrm{H_2O}$)**.** Let the atomic base set be

$$V_0 = \{H_1,\ H_2,\ Na,\ Cl,\ O\}.$$

Level–1 (simple) hyperedges encode intramolecular bonds in reactants/products:

$$E_S = \big\{\{H_1, Cl\},\ \{Na, O\},\ \{O, H_2\},\ \{Na, Cl\},\ \{O, H_1\}\big\}.$$

Molecule (nesting) hyperedges are subsets of E_S:

$$M_{\mathrm{HCl}} = \big\{\{H_1, Cl\}\big\}, \quad M_{\mathrm{NaOH}} = \big\{\{Na, O\}, \{O, H_2\}\big\},$$
$$M_{\mathrm{NaCl}} = \big\{\{Na, Cl\}\big\}, \quad M_{\mathrm{H_2O}} = \big\{\{O, H_1\}, \{O, H_2\}\big\}.$$

To encode the overall reaction as a single directed hyperedge at the same level, collect reactant and product molecules into "complex" hyperedges

$$M_{\mathrm{R}} = M_{\mathrm{HCl}} \cup M_{\mathrm{NaOH}}, \qquad M_{\mathrm{P}} = M_{\mathrm{NaCl}} \cup M_{\mathrm{H_2O}},$$

and set

$$E_N = \big\{M_{\mathrm{HCl}},\ M_{\mathrm{NaOH}},\ M_{\mathrm{NaCl}},\ M_{\mathrm{H_2O}},\ M_{\mathrm{R}},\ M_{\mathrm{P}}\big\},$$
$$E_D = \big\{(M_{\mathrm{R}} \to M_{\mathrm{P}})\big\}.$$

Then $\mathcal{H} = (V_0, E_S, E_N, E_D)$ is a chemical hypergraph with atoms V_0, bonds/substructures E_S, molecules/complexes E_N, and a directed reaction hyperedge E_D.

Definition 10 (m-Multilevel n-SuperHypergraph)**.** Let V_0 be a non-empty finite set of base nodes. For $n \geq 0$, define the n-th iterated powerset recursively as

$$\mathcal{P}^0(V_0) = V_0, \quad \mathcal{P}^{k+1}(V_0) = \mathcal{P}(\mathcal{P}^k(V_0)).$$

An *m-Multilevel n-SuperHypergraph* is a tuple

$$\mathcal{M}^{(m,n)} = \big(V_0,\ E_1, \ldots, E_m,\ w_1, \ldots, w_m\big),$$

where:

- For each $1 \leq \ell \leq m$, $E_\ell \subseteq \mathcal{P}^n(V_0)$ is the set of level-ℓ hyperedges, each consisting of elements of $\mathcal{P}^n(V_0)$.
- $w_\ell : E_\ell \to \mathbb{R}_{\geq 0}$ is an optional weight function assigning a non-negative real value to each level-ℓ hyperedge.
- The levels are hierarchical: for each $\ell \geq 2$, hyperedges in E_ℓ are (nonempty) subsets of $E_{\ell-1}$.

In other words:

$$E_1 \subseteq \mathcal{P}^n(V_0), \quad E_2 \subseteq \mathcal{P}(E_1),$$
$$E_3 \subseteq \mathcal{P}(E_2), \quad \ldots, \quad E_m \subseteq \mathcal{P}(E_{m-1}).$$

Definition 11 (Chemical SuperHyperGraph)**.** A *chemical superhypergraph* is an m-Multilevel n-SuperHypergraph

$$\mathcal{CSH}^{(m,n)} = \big(V_0,\ E_1,\ E_2,\ \ldots,\ E_m,\ w_1,\ \ldots,\ w_m\big)$$

equipped with the following chemical interpretations:

- V_0 is the set of *atoms*.
- $E_1 \subseteq \mathcal{P}^n(V_0)$ is the set of *simple bonds or functional groups*, each a (possibly fuzzy) subset of $\mathcal{P}^n(V_0)$.
- $E_2 \subseteq \mathcal{P}(E_1)$ is the set of *molecular substructures*, each a nonempty collection of simple bonds.
- $\ldots$
- $E_n \subseteq \mathcal{P}(E_{n-1})$ is the set of *complete molecules*, each grouping lower-level substructures.
- $E_{n+1}, \ldots, E_m$ may represent higher-order reaction networks, phase transitions, or functional aggregates.

- w_ℓ can encode bond strengths, reaction rates, or other quantitative chemical properties.

Example 6 (Chemical SuperHypergraph: Ethanol within a Binary Mixture)**.** Let $m = 3$, $n = 1$ and take the atomic base set

$$V_0 = \{C_1, C_2, O_1, H_a, H_b, H_c, H_d, H_e, H_f\} \cup \{O_w, H_{w1}, H_{w2}\},$$

representing ethanol $(C_1, C_2, O_1, H_a, \ldots, H_f)$ and water (O_w, H_{w1}, H_{w2}).

Level 1 hyperedges $E_1 \subseteq \mathcal{P}^1(V_0) = \mathcal{P}(V_0)$ encode bonds (2–sets):

$$E_1^{\mathrm{EtOH}} = \big\{\{C_1, C_2\},\ \{C_2, O_1\},\ \{C_1, H_a\},\ \{C_1, H_b\},\ \{C_1, H_c\},\ \{C_2, H_d\},\ \{C_2, H_e\},\ \{O_1, H_f\}\big\},$$
$$E_1^{\mathrm{H_2O}} = \big\{\{O_w, H_{w1}\},\ \{O_w, H_{w2}\}\big\},$$

and set $E_1 = E_1^{\mathrm{EtOH}} \cup E_1^{\mathrm{H_2O}}$. (Optionally, $w_1 : E_1 \to \{1\}$ stores single–bond order.)

Level 2 hyperedges group bonds into molecules (elements of $\mathcal{P}(E_1)$):

$$E_2 = \big\{M_{\mathrm{EtOH}} := E_1^{\mathrm{EtOH}},\quad M_{\mathrm{H_2O}} := E_1^{\mathrm{H_2O}}\big\}.$$

Level 3 hyperedges group molecules into a mixture (elements of $\mathcal{P}(E_2)$):

$$E_3 = \big\{\ \mathsf{Mixture} := \{M_{\mathrm{EtOH}},\ M_{\mathrm{H_2O}}\}\ \big\}.$$

Thus the chemical superhypergraph is

$$\mathcal{CSH}^{(3,1)} = \big(V_0,\ E_1,\ E_2,\ E_3,\ w_1\big),$$

where level–1 encodes bonds, level–2 encodes whole molecules, and level–3 encodes the binary mixture as a nested aggregate.

Example 7 (Chemical SuperHypergraph: Tetraamminecopper(II) Complex with Hydration Shell)**.** Let $m = 4$, $n = 1$. The atomic base set is

$$V_0 = \Big\{\mathrm{Cu}\Big\} \cup \bigcup_{i=1}^{4}\Big\{N_i, H_{i1}, H_{i2}, H_{i3}\Big\} \cup \bigcup_{j=1}^{2}\Big\{O_j^{(w)}, H_{j1}^{(w)}, H_{j2}^{(w)}\Big\},$$

representing Cu^{2+}, four *ammine* ligands NH_3, and two water molecules.

Level 1 (bonds) $E_1 \subseteq \mathcal{P}(V_0)$:

$$E_1^{\mathrm{Cu\text{–}N}} = \big\{\{\mathrm{Cu}, N_i\} : i = 1, 2, 3, 4\big\},$$
$$E_1^{\mathrm{NH_3}} = \big\{\{N_i, H_{i\alpha}\} : i = 1, 2, 3, 4,\ \alpha = 1, 2, 3\big\},$$
$$E_1^{\mathrm{H_2O}} = \big\{\{O_j^{(w)}, H_{jk}^{(w)}\} : j = 1, 2,\ k = 1, 2\big\},$$
$$E_1 = E_1^{\mathrm{Cu\text{–}N}} \cup E_1^{\mathrm{NH_3}} \cup E_1^{\mathrm{H_2O}}.$$

Level 2 (molecular substructures) $E_2 \subseteq \mathcal{P}(E_1)$:

$$L_i = \big\{\{N_i, H_{i1}\}, \{N_i, H_{i2}\}, \{N_i, H_{i3}\}, \{\mathrm{Cu}, N_i\}\big\} \quad (i = 1, \ldots, 4),$$
$$W_j = \big\{\{O_j^{(w)}, H_{j1}^{(w)}\}, \{O_j^{(w)}, H_{j2}^{(w)}\}\big\} \quad (j = 1, 2), \qquad E_2 = \{L_1, L_2, L_3, L_4, W_1, W_2\}.$$

Level 3 (complex vs. solvation) $E_3 \subseteq \mathcal{P}(E_2)$:

$$\mathsf{Complex} = \{L_1, L_2, L_3, L_4\}, \qquad \mathsf{Hydration} = \{W_1, W_2\}, \qquad E_3 = \{\mathsf{Complex}, \mathsf{Hydration}\}.$$

Level 4 (system aggregate) $E_4 \subseteq \mathcal{P}(E_3)$:

$$\mathsf{System} = \{\mathsf{Complex}, \mathsf{Hydration}\}, \qquad E_4 = \{\mathsf{System}\}.$$

The chemical superhypergraph is $\mathcal{CSH}^{(4,1)} = \big(V_0, E_1, E_2, E_3, E_4\big)$, with bonds (level 1), ligands/waters (level 2), complex/solvation (level 3), and the overall system (level 4).

Example 8 (Chemical SuperHypergraph: Acid–Alcohol Esterification)**.** Consider the (un-catalyzed) rearrangement $CH_3COOH + CH_3CH_2OH \rightleftharpoons CH_3COOCH_2CH_3 + H_2O$. Let $m = 4$, $n = 1$ and use a single atomic base set V_0 shared by both sides:

$$\begin{aligned} V_0 = {} & \{C_1, C_2, O_1, O_2, H_a, H_b, H_c, H_{oh}\} \ \text{(acetic acid atoms)} \\ & \cup \ \{C_3, C_4, O_3, H_1, H_2, H_3, H_4, H_5, H'_{oh}\} \ \text{(ethanol atoms)}. \end{aligned}$$

Level 1 (bonds used across both sides) $E_1 \subseteq \mathcal{P}(V_0)$:

$$\begin{aligned} E_1^{\mathrm{AcOH}} = {} & \{\{C_1, H_a\},\ \{C_1, H_b\},\ \{C_1, H_c\},\ \{C_1, C_2\}, \\ & \{C_2, O_1\},\ \{C_2, O_2\},\ \{O_2, H_{oh}\}\}, \end{aligned}$$

$$\begin{aligned} E_1^{\mathrm{EtOH}} = {} & \{\{C_3, H_1\},\ \{C_3, H_2\},\ \{C_3, H_3\},\ \{C_3, C_4\}, \\ & \{C_4, H_4\},\ \{C_4, H_5\},\ \{C_4, O_3\},\ \{O_3, H'_{oh}\}\}, \end{aligned}$$

$$\begin{aligned} E_1^{\mathrm{EtOAc}} = {} & \{\{C_1, H_a\},\ \{C_1, H_b\},\ \{C_1, H_c\},\ \{C_1, C_2\}, \\ & \{C_2, O_1\},\ \{C_2, O_2\},\ \{O_2, C_4\},\ \{C_3, H_1\}, \\ & \{C_3, H_2\},\ \{C_3, H_3\},\ \{C_3, C_4\},\ \{C_4, H_4\},\ \{C_4, H_5\}\}, \end{aligned}$$

$$E_1^{\mathrm{H_2O}} = \{\{O_3, H_{oh}\},\ \{O_3, H'_{oh}\}\},$$

$$E_1 = E_1^{\mathrm{AcOH}} \ \cup \ E_1^{\mathrm{EtOH}} \ \cup \ E_1^{\mathrm{EtOAc}} \ \cup \ E_1^{\mathrm{H_2O}}.$$

(Edges that appear in multiple sets are identified set-theoretically; optional weights w_1 can store single/double bond orders.)

Level 2 (molecules as sets of bonds) $E_2 \subseteq \mathcal{P}(E_1)$:

$$M_{\mathrm{AcOH}} = E_1^{\mathrm{AcOH}}, \qquad M_{\mathrm{EtOH}} = E_1^{\mathrm{EtOH}}, \qquad M_{\mathrm{EtOAc}} = E_1^{\mathrm{EtOAc}}, \qquad M_{\mathrm{H_2O}} = E_1^{\mathrm{H_2O}},$$
$$E_2 = \{M_{\mathrm{AcOH}}, M_{\mathrm{EtOH}}, M_{\mathrm{EtOAc}}, M_{\mathrm{H_2O}}\}.$$

Level 3 (reactant/product complexes) $E_3 \subseteq \mathcal{P}(E_2)$:

$$\mathsf{Reactants} = \{M_{\mathrm{AcOH}}, M_{\mathrm{EtOH}}\}, \qquad \mathsf{Products} = \{M_{\mathrm{EtOAc}}, M_{\mathrm{H_2O}}\}, \qquad E_3 = \{\mathsf{Reactants}, \mathsf{Products}\}.$$

Level 4 (reaction macro-unit) $E_4 \subseteq \mathcal{P}(E_3)$:

$$\mathsf{Esterification} = \{\mathsf{Reactants}, \mathsf{Products}\}, \qquad E_4 = \{\mathsf{Esterification}\}.$$

Hence, $\mathcal{CSH}^{(4,1)} = \big(V_0, E_1, E_2, E_3, E_4\big)$ is a chemical superhypergraph with levels for bonds (1), molecules (2), side-specific complexes (3), and the overall chemical transformation (4).

2 | Main Results

In this section, we introduce several new graph concepts and provide their formal definitions.

2.1 | Electrochemical Graph

Electrochemistry investigates relationships between electrical energy and chemical reactions, including redox processes, batteries, fuel cells, corrosion, and electrolysis [24, 25, 26]. An *Electrochemical Graph* models ions, electrons, and redox processes as nodes and edges, capturing electrochemical reactions with stoichiometric balance constraints.

Definition 12 (Electrochemical Graph)**.** An *electrochemical graph* is a typed, directed, bipartite multigraph

$$\mathcal{G}_{\mathrm{ec}} = (S, P, E, \mathrm{src}, \mathrm{tgt}, \mathrm{typ}_S, \mathrm{typ}_P, \nu, q, \mathrm{comp}),$$

where:

- S is a finite set of *species nodes* (ions, molecules, electrons).
- P is a finite set of *process nodes* (electron–transfer, redox half–reactions, adsorption, transport).
- $E \subseteq (S \times P) \cup (P \times S)$ is a finite set of directed edges with source/target maps $\mathrm{src}, \mathrm{tgt}$.
- $\mathrm{typ}_S : S \to \{\mathsf{Ion}, \mathsf{Molecule}, \mathsf{Electron}\}$ and $\mathrm{typ}_P : P \to \{\mathsf{ET}, \mathsf{Redox}, \mathsf{Adsorb}, \mathsf{Transport}\}$ are typing maps.
- $\nu : E \to \mathbb{N}$ assigns a *stoichiometric coefficient* to each edge. We write $\sigma(e) = -1$ if e goes $S \to P$ (consumed) and $\sigma(e) = +1$ if e goes $P \to S$ (produced).
- $q : S \to \mathbb{Z}$ is the *charge* map (with $q(e^-) = -1$ for the electron species).
- $\mathrm{comp} : S \to \mathcal{C}$ assigns a *compartment/phase* (e.g. anode, cathode, electrolyte).

The following *conservation constraint* holds for every process $p \in P$:

$$\sum_{\substack{e \in E \\ \mathrm{tgt}(e)=p \text{ or } \mathrm{src}(e)=p}} \sigma(e)\,\nu(e)\,q\big(s_e\big) = 0, \qquad s_e \in S \text{ the species incident to } e,$$

i.e. net charge is balanced at each process. Additional invariants (e.g. element balance) can be encoded by replacing q with vector-valued conserved quantities.

Example 9 (Electrochemical Graph: Fe^{3+}/Fe^{2+} Redox Step)**.** Let the species set be

$$S = \{\mathrm{Fe}^{3+},\ \mathrm{Fe}^{2+},\ e^-\}, \quad q(\mathrm{Fe}^{3+}) = +3,\ q(\mathrm{Fe}^{2+}) = +2,\ q(e^-) = -1.$$

Introduce one process node p_{red} (reduction) and directed edges with stoichiometries

$$\mathrm{Fe}^{3+} \xrightarrow[\nu=1]{} p_{\mathrm{red}}, \qquad e^- \xrightarrow[\nu=1]{} p_{\mathrm{red}}, \qquad p_{\mathrm{red}} \xrightarrow[\nu=1]{} \mathrm{Fe}^{2+}.$$

Charge conservation at p_{red} holds:

$$(-1)\cdot 1\cdot q(\mathrm{Fe}^{3+}) + (-1)\cdot 1\cdot q(e^-) + (+1)\cdot 1\cdot q(\mathrm{Fe}^{2+}) = -3 + 1 + 2 = 0.$$

This realizes the half–reaction $\mathrm{Fe}^{3+} + e^- \to \mathrm{Fe}^{2+}$ in the electrochemical graph formalism.

Definition 13 (Electrochemical Hypergraph)**.** Let S be a finite set of *species* (including the electron $e^- \in S$) and let $q : S \to \mathbb{Z}$ be the *charge map* (with $q(e^-) = -1$). An *electrochemical hypergraph* is a tuple

$$\mathcal{H}_{\mathrm{ec}} = \big(S, \mathcal{E}_{\mathrm{redox}}, \alpha, \beta, \mathrm{comp}, w\big),$$

where:

- $\mathcal{E}_{\mathrm{redox}}$ is a finite set of *directed hyperedges* (redox/ET steps);
- for each $h \in \mathcal{E}_{\mathrm{redox}}$, $\alpha_h, \beta_h : S \to \mathbb{N}_0$ are *reactant/product stoichiometry maps* with finite supports $\mathrm{supp}(\alpha_h) = \{s : \alpha_h(s) > 0\}$ and $\mathrm{supp}(\beta_h) = \{s : \beta_h(s) > 0\}$, not both empty;
- $\mathrm{comp} : S \to \mathcal{C}$ assigns a *compartment/phase* (e.g., anode, cathode, electrolyte);
- $w : \mathcal{E}_{\mathrm{redox}} \to \mathbb{R}_{\geq 0}$ is an optional weight (e.g., rate constant or propensity).

Each $h \in \mathcal{E}_{\mathrm{redox}}$ must satisfy charge conservation

$$\sum_{s \in S} \big(\beta_h(s) - \alpha_h(s)\big)\, q(s) = 0,$$

and, if required, additional linear conservation laws (e.g., elemental balances) by replacing q with vector-valued conserved quantities.

Example 10 (Electrochemical HyperGraph: Oxygen Reduction Reaction (ORR))**.** Let the vertex set be

$$V = \{\mathrm{H}^+,\ e^-,\ \mathrm{O}_2,\ \mathrm{H}_2\mathrm{O},\ \mathsf{Cathode}\}.$$

Define one typed hyperedge $E = \{h_{\mathrm{ORR}}\}$ with

$$h_{\mathrm{ORR}} = \big(\mathsf{Reaction},\ \{\mathrm{H}^+,\ e^-,\ \mathrm{O}_2,\ \mathrm{H}_2\mathrm{O},\ \mathsf{Cathode}\}\big),$$

and attach stoichiometry/sign via a map $\nu_{\mathrm{ORR}} : V \to \mathbb{Z}$:

$$\nu_{\mathrm{ORR}}(\mathrm{H}^+) = -2, \quad \nu_{\mathrm{ORR}}(e^-) = -2, \quad \nu_{\mathrm{ORR}}(\mathrm{O}_2) = -1, \quad \nu_{\mathrm{ORR}}(\mathrm{H_2O}) = +2, \quad \nu_{\mathrm{ORR}}(\mathsf{Cathode}) = 0.$$

This hyperedge encodes the multiway reaction $2\,\mathrm{H}^+ + 2\,e^- + \frac{1}{2}\mathrm{O}_2 \to \mathrm{H_2O}$ (aggregated here as integral stoichiometry $-2, -2, -1, +2$), anchored to the Cathode.

Definition 14 (Electrochemical SuperHyperGraph)**.** Let S_0 be a finite set of *chemical species* that includes the electron e^-, and let $q : S_0 \to \mathbb{Z}$ be the *charge map* (with $q(e^-) = -1$). Fix $n \in \mathbb{N}_0$ and choose a finite set of n–supervertices $V \subseteq \mathcal{P}^n(S_0)$. An *electrochemical superhypergraph* is a tuple

$$\mathcal{H}_{\mathrm{ec}}^{(n)} = (S_0, V, \mathcal{E}, \alpha, \beta, \mathrm{comp}, w),$$

where:

- $\mathcal{E}$ is a finite set of *directed superhyperedges* (redox/ET steps);
- for each $h \in \mathcal{E}$, $\alpha_h, \beta_h : V \to \mathbb{N}_0$ are superstoichiometries with finite supports;
- $\mathrm{comp} : S_0 \to \mathcal{C}$ assigns a compartment/phase (e.g. anode, cathode, electrolyte);
- $w : \mathcal{E} \to \mathbb{R}_{\geq 0}$ is an optional weight (rate/propensity).

The *induced base-level stoichiometries* are

$$\widehat{\alpha}_h(s) := \sum_{A \in V} \alpha_h(A)\, \iota_n(A, s), \qquad \widehat{\beta}_h(s) := \sum_{A \in V} \beta_h(A)\, \iota_n(A, s) \quad (s \in S_0).$$

Each $h \in \mathcal{E}$ satisfies *charge conservation*

$$\sum_{s \in S_0} \big(\widehat{\beta}_h(s) - \widehat{\alpha}_h(s)\big)\, q(s) = 0,$$

and, if required, additional linear conservation laws (e.g. elemental balances) by replacing q with a vector of conserved counts.

Example 11 (Electrochemical SuperHyperGraph: Two-Stage Redox at a Cathode)**.** Let the base set of electrochemical atoms be

$$V_0 = \{\mathrm{Fe}^{3+},\ \mathrm{Fe}^{2+},\ e^-,\ \mathsf{Cathode},\ \mathsf{Electrolyte}\}.$$

Elements of $\mathcal{P}^2(V_0)$ are collections of nonempty subsets of V_0. Define three 2–supervertices

$$A_1 = \big\{\{\mathrm{Fe}^{3+}, e^-\},\ \{\mathrm{Fe}^{2+}\}\big\},$$
$$A_2 = \big\{\{\mathsf{Cathode}\},\ \{e^-\}\big\},$$
$$A_3 = \big\{\{\mathsf{Electrolyte}\},\ \{\mathrm{Fe}^{3+}\},\ \{\mathrm{Fe}^{2+}\}\big\}.$$

Set $V = \{A_1, A_2, A_3\} \subseteq \mathcal{P}^2(V_0)$ and

$$E = \big\{\{A_1, A_2\},\ \{A_2, A_3\}\big\} \subseteq \mathcal{P}(V) \setminus \{\emptyset\}.$$

Then $\mathrm{SHG}^{(2)} = (V, E)$ is a 2–SuperHyperGraph encoding a half–reaction group (A_1), an electrode–interface group (A_2), and an environment group (A_3), with superedges capturing their coupling.

2.2 | Biochemical Graph

Biochemistry studies chemical processes within living organisms, focusing on biomolecules such as proteins, nucleic acids, carbohydrates, and lipids [27, 28, 29]. A *Biochemical Graph* represents metabolites, enzymes, and reactions as nodes, with stoichiometric and regulatory edges modeling biochemical pathways and cellular processes.

Definition 15 (Biochemical Graph)**.** A *biochemical graph* is a typed, directed, bipartite multigraph

$$\mathcal{G}_{\mathrm{bio}} = (S, P, E_{\mathrm{st}}, E_{\mathrm{reg}}, \mathrm{src}, \mathrm{tgt}, \mathrm{typ}_S, \mathrm{typ}_P, \nu, \mathrm{comp}),$$

with:

- S a finite set of *species* (metabolites, proteins/enzymes, genes).
- P a finite set of *processes* (biochemical reactions; transcription/translation modules).
- $E_{\mathrm{st}} \subseteq (S \times P) \cup (P \times S)$ *stoichiometric edges*; $E_{\mathrm{reg}} \subseteq (S \times P)$ *regulatory edges* (activation/inhibition).
- $\mathrm{typ}_S : S \to \{\mathsf{Metabolite}, \mathsf{Protein}, \mathsf{Gene}\}$ and $\mathrm{typ}_P : P \to \{\mathsf{Reaction}, \mathsf{Regulation}\}$.
- $\nu : E_{\mathrm{st}} \to \mathbb{N}$ stoichiometric coefficients with sign via direction as in the electrochemical case.
- $\mathrm{comp} : S \to \mathcal{C}$ assigns cellular compartments (e.g. cytosol, mitochondrion).

Mass-balance at each reaction $p \in P$ is expressed by conservation of elemental counts: for any conserved vector $c : \mathsf{Metabolite} \to \mathbb{Z}^d$ (e.g. atoms of C,H,N,O,...),

$$\sum_{e \in E_{\mathrm{st}} : p \text{ incident}} \sigma(e)\, \nu(e)\, c\big(s_e\big) = 0.$$

Regulatory edges in E_{reg} do not carry stoichiometry; they modulate kinetics or enable/disable p.

Example 12 (Biochemical Graph: Hexokinase Step in Glycolysis)**.** Let metabolite nodes be

$$S_{\mathrm{met}} = \{\mathrm{Glc},\ \mathrm{ATP},\ \mathrm{G6P},\ \mathrm{ADP}\},$$

and one enzyme node HK (hexokinase). Introduce a reaction node p_{HK} and stoichiometric edges

$$\mathrm{Glc} \xrightarrow[\nu=1]{} p_{\mathrm{HK}}, \quad \mathrm{ATP} \xrightarrow[\nu=1]{} p_{\mathrm{HK}},$$

$$p_{\mathrm{HK}} \xrightarrow[\nu=1]{} \mathrm{G6P}, \quad p_{\mathrm{HK}} \xrightarrow[\nu=1]{} \mathrm{ADP}.$$

Add a regulatory edge $\mathrm{HK} \to p_{\mathrm{HK}}$ (activation). Phosphate-atom balance checks:

$$\underbrace{P(\mathrm{ATP})}_{3} = \underbrace{P(\mathrm{ADP})}_{2} + \underbrace{P(\mathrm{G6P})}_{1} \ \Rightarrow\ 3 = 2 + 1.$$

Thus the biochemical graph captures $\mathrm{Glc} + \mathrm{ATP} \to \mathrm{G6P} + \mathrm{ADP}$ with enzymatic control.

Definition 16 (Biochemical Hypergraph)**.** Let S be a finite set of biological species with a typing $\mathrm{typ}_S : S \to \{\mathsf{Metabolite}, \mathsf{Protein}, \mathsf{Gene}\}$. A *biochemical hypergraph* is

$$\mathcal{H}_{\mathrm{bio}} = \big(S, \mathcal{E}_{\mathrm{rxn}}, \mathcal{E}_{\mathrm{reg}}, \alpha, \beta, R^+, R^-, \mathrm{comp}, w\big),$$

where:

- $\mathcal{E}_{\mathrm{rxn}}$ is a finite set of *reaction hyperedges*; for $h \in \mathcal{E}_{\mathrm{rxn}}$, $\alpha_h, \beta_h : S \to \mathbb{N}_0$ are reactant/product stoichiometry with finite supports;
- $\mathcal{E}_{\mathrm{reg}}$ is a finite set of *regulatory hyperedges*; each $r \in \mathcal{E}_{\mathrm{reg}}$ is a pair (R_r^+, R_r^-) with $R_r^{\pm} \subseteq S$ (activators/inhibitors) that *reference* a unique $h \in \mathcal{E}_{\mathrm{rxn}}$ (notationally encoded via $w(r,h) > 0$ if desired);
- $R^+, R^- : \mathcal{E}_{\mathrm{rxn}} \to \mathcal{P}(S)$ optionally attach sets of embedded activators/inhibitors to each reaction h;
- $\mathrm{comp} : S \to \mathcal{C}$ assigns cellular compartments;
- w assigns optional weights to reactions/regulations.

For each reaction $h \in \mathcal{E}_{\mathrm{rxn}}$ and any conserved composition map $c : \{s \in S : \mathrm{typ}_S(s) = \mathsf{Metabolite}\} \to \mathbb{Z}^d$ (e.g., atomic counts),

$$\sum_{s \in S} \big(\beta_h(s) - \alpha_h(s)\big)\, c(s) = 0.$$

Regulatory hyperedges constrain feasibility/kinetics of h but carry no stoichiometric balance themselves.

Example 13 (Biochemical HyperGraph: Citrate Synthase Step)**.** Let

$$V = \{\text{AcCoA, OAA, } \text{H}_2\text{O, Citrate, CoA, } \mathsf{CS}\}.$$

Introduce one hyperedge $h_{\text{CS}} = (\mathsf{Reaction}, \{\text{AcCoA}, \text{OAA}, \text{H}_2\text{O}, \text{Citrate}, \text{CoA}, \mathsf{CS}\})$ with stoichiometry

$$\nu_{\text{CS}}(\text{AcCoA}) = -1,\ \nu_{\text{CS}}(\text{OAA}) = -1,\ \nu_{\text{CS}}(\text{H}_2\text{O}) = -1,$$
$$\nu_{\text{CS}}(\text{Citrate}) = +1,\ \nu_{\text{CS}}(\text{CoA}) = +1,\ \nu_{\text{CS}}(\mathsf{CS}) = 0,$$

capturing $\text{AcCoA} + \text{OAA} + \text{H}_2\text{O} \to \text{Citrate} + \text{CoA}$ with enzyme CS participating catalytically.

Definition 17 (Biochemical SuperHyperGraph)**.** Let S_0 be a finite set of biological species with typing $\text{typ}_S : S_0 \to \{\mathsf{Metabolite}, \mathsf{Protein}, \mathsf{Gene}\}$, and let $\text{comp} : S_0 \to \mathcal{C}$ assign cellular compartments. Fix $n \in \mathbb{N}_0$ and choose $V \subseteq \mathcal{P}^n(S_0)$ finite. A *biochemical superhypergraph* is

$$\mathcal{H}^{(n)}_{\text{bio}} = (S_0, V, \mathcal{E}_{\text{rxn}}, \mathcal{E}_{\text{reg}}, \alpha, \beta, R^+, R^-, w),$$

where:

- $\mathcal{E}_{\text{rxn}}$ are *reaction* superhyperedges with $\alpha_h, \beta_h : V \to \mathbb{N}_0$ (finite support);
- $\mathcal{E}_{\text{reg}}$ are *regulatory* superhyperedges; for $r \in \mathcal{E}_{\text{reg}}$ that references $h \in \mathcal{E}_{\text{rxn}}$, one assigns $R^+(r), R^-(r) \subseteq V$ (activator/inhibitor supervertex sets);
- w provides optional weights (evidence/strength).

With induced base stoichiometries $\widehat{\alpha}_h, \widehat{\beta}_h$ as above, every $h \in \mathcal{E}_{\text{rxn}}$ satisfies, for any conserved composition $c : \{s \in S_0 : \text{typ}_S(s) = \mathsf{Metabolite}\} \to \mathbb{Z}^d$,

$$\sum_{s \in S_0} \big(\widehat{\beta}_h(s) - \widehat{\alpha}_h(s)\big)\, c(s) = 0.$$

Regulatory superhyperedges constrain feasibility/kinetics but carry no stoichiometry.

Example 14 (Biochemical SuperHyperGraph: Hexokinase Stage and Context)**.** Let

$$V_0 = \{\text{Glc, ATP, G6P, ADP, } \mathsf{HK}, \mathsf{Cytosol}\}.$$

Define

$$A_1 = \big\{\{\text{Glc}, \text{ATP}\}, \{\text{G6P}, \text{ADP}\}\big\},$$
$$A_2 = \big\{\{\mathsf{HK}\}, \{\mathsf{Cytosol}\}\big\},$$
$$A_3 = \big\{\{\text{G6P}\}, \{\mathsf{HK}\}\big\}.$$

With $V = \{A_1, A_2, A_3\}$ and $E = \big\{\{A_1, A_2\}, \{A_1, A_3\}\big\}$, the 2–SuperHyperGraph $\text{SHG}^{(2)} = (V, E)$ groups reactants/products (A_1), enzyme/location context (A_2), and product–enzyme feedback (A_3), and links them via superedges.

2.3 | Electrobiochemical Graph

Electrobiochemistry studies interactions of electrical phenomena with biochemical systems, investigating electron transfer, bioelectrochemical reactions, biosensors, bioenergetics, and electrically driven cellular processes[30, 31, 32, 33, 34]. An *Electrobiochemical Graph* integrates biochemical and electrochemical components, combining redox electron transfer, enzymes, metabolites, and reactions into a unified structural representation.

Definition 18 (Electrobiochemical Graph)**.** An *electrobiochemical graph* integrates electrochemistry and biochemistry in a single structure

$$\mathcal{G}_{\text{eb}} = (S, P, E_{\text{st}}, E_{\text{reg}}, \text{src}, \text{tgt}, \text{typ}_S, \text{typ}_P, \nu, q, \text{comp}),$$

where the components are as in the previous two definitions, with combined typings

$$\text{typ}_S : S \to \{\mathsf{Metabolite}, \mathsf{Protein}, \mathsf{Gene}, \mathsf{Ion}, \mathsf{Electron}\},$$
$$\text{typ}_P : P \to \{\mathsf{Reaction}, \mathsf{ET}, \mathsf{Transport}, \mathsf{Regulation}\}.$$

Stoichiometric and regulatory edges are defined as before; electrons are explicit species ($q(e^-) = -1$). Each process node $p \in P$ satisfies *simultaneous* charge and elemental balances:

$$\sum_{e \in E_{\mathrm{st}} : p \text{ incident}} \sigma(e)\, \nu(e)\, q(s_e) = 0,$$

$$\sum_{e \in E_{\mathrm{st}} : p \text{ incident}} \sigma(e)\, \nu(e)\, c(s_e) = 0,$$

for any conserved composition vector c (e.g. atomic counts). This allows redox biocatalysis and electrode–enzyme electron exchange to be represented coherently.

Example 15 (Electrobiochemical Graph: Formate Dehydrogenase with Electrode ET)**.** Species:

$$S = \{\mathrm{HCOO^-},\ \mathrm{CO_2},\ \mathrm{NAD^+},\ \mathrm{NADH},\ \mathrm{H^+},\ e^-\},$$

$$q(\mathrm{HCOO^-}) = -1,\ q(\mathrm{NAD^+}) = +1,\ q(\mathrm{H^+}) = +1,\ q(e^-) = -1.$$

Processes: a biochemical reaction p_1 and an electron–transfer (ET) step p_2. Stoichiometric edges for p_1 (formate dehydrogenase):

$$\mathrm{HCOO^-} \xrightarrow[\nu=1]{} p_1, \quad \mathrm{NAD^+} \xrightarrow[\nu=1]{} p_1,$$

$$p_1 \xrightarrow[\nu=1]{} \mathrm{CO_2}, \quad p_1 \xrightarrow[\nu=1]{} \mathrm{NADH}.$$

This reaction is charge balanced: $(-1) + (+1) = 0$ on reactants, $0 + 0 = 0$ on products. ET step p_2 (to an electrode mediator):

$$\mathrm{NADH} \xrightarrow[\nu=1]{} p_2, \qquad p_2 \xrightarrow[\nu=1]{} \mathrm{NAD^+},$$

$$p_2 \xrightarrow[\nu=1]{} \mathrm{H^+}, \quad p_2 \xrightarrow[\nu=2]{} e^-.$$

Charge balance at p_2: $0 = (+1) + (+1) + 2(-1) = 0$. This graph coherently integrates redox biochemistry and explicit electron transfer.

Definition 19 (Electrobiochemical Hypergraph)**.** An *electrobiochemical hypergraph* augments the biochemical hypergraph by including charged species and electrons. Let

$$\mathcal{H}_{\mathrm{eb}} = \big(S, \mathcal{E}_{\mathrm{proc}}, \alpha, \beta, R^+, R^-, q, \mathrm{comp}, w\big),$$

where S includes Ion and $\mathsf{Electron}$ types in addition to $\mathsf{Metabolite}, \mathsf{Protein}, \mathsf{Gene}$, and $q : S \to \mathbb{Z}$ is the charge map ($q(e^-) = -1$). Each process hyperedge $h \in \mathcal{E}_{\mathrm{proc}}$ (with stoichiometry α_h, β_h) must satisfy simultaneously

$$\sum_{s \in S} \big(\beta_h(s) - \alpha_h(s)\big)\, q(s) = 0$$

$$\text{and} \quad \sum_{s \in S} \big(\beta_h(s) - \alpha_h(s)\big)\, c(s) = 0,$$

for any conserved elemental map c. Regulatory sets $R^{\pm}(h) \subseteq S$ optionally encode activation/inhibition of h.

Example 16 (Electrobiochemical HyperGraph: NADH Oxidation at an Electrode)**.** Vertices

$$V = \{\mathrm{NADH},\ \mathrm{NAD^+},\ \mathrm{H^+},\ e^-,\ \mathsf{Anode},\ \mathsf{FDH}\}.$$

Define two hyperedges:

$$h_1 = (\mathsf{Bioreaction},\ \{\mathrm{NAD^+},\ \mathrm{NADH},\ \mathsf{FDH}\}),$$

$$h_2 = (\mathsf{ET},\ \{\mathrm{NADH},\ \mathrm{NAD^+},\ \mathrm{H^+},\ e^-,\ \mathsf{Anode}\}).$$

With stoichiometries

$$\nu_1(\mathrm{NAD^+}) = -1,\ \nu_1(\mathrm{NADH}) = +1,\ \nu_1(\mathsf{FDH}) = 0,$$

and

$$\nu_2(\mathrm{NADH}) = -1,\ \nu_2(\mathrm{NAD^+}) = +1,\ \nu_2(\mathrm{H^+}) = +1,\ \nu_2(e^-) = +2,\ \nu_2(\mathsf{Anode}) = 0,$$

the pair $\{h_1, h_2\}$ encodes enzyme-coupled redox and the electron-transfer step to the Anode.

Definition 20 (Electrobiochemical SuperHyperGraph)**.** Let S_0 contain typed Metabolite/Protein/Gene as well as Ion and Electron species, with a charge map $q : S_0 \to \mathbb{Z}$ ($q(e^-)=-1$). Fix n and $V \subseteq \mathcal{P}^n(S_0)$ finite. An *electrobiochemical superhypergraph* is

$$\mathcal{H}_{\mathrm{eb}}^{(n)} = (S_0, V, \mathcal{E}_{\mathrm{proc}}, \mathcal{E}_{\mathrm{reg}}, \alpha, \beta, R^+, R^-, w),$$

where $\mathcal{E}_{\mathrm{proc}}$ are process superhyperedges (reactions, ET, transport) with superstoichiometries α_h, β_h. For each $h \in \mathcal{E}_{\mathrm{proc}}$ the induced base-level balances hold:

$$\sum_{s \in S_0} \big(\widehat{\beta}_h(s) - \widehat{\alpha}_h(s)\big)\, q(s) = 0,$$

$$\sum_{s \in S_0} \big(\widehat{\beta}_h(s) - \widehat{\alpha}_h(s)\big)\, c(s) = 0,$$

for any conserved elemental map c. Regulatory sets $R^{\pm}$ are as in the biochemical case.

Example 17 (Electrochemical SuperHyperGraph: Two-Stage Redox at a Cathode)**.** Let the base set of electrochemical atoms be

$$V_0 = \{\mathrm{Fe}^{3+},\ \mathrm{Fe}^{2+},\ e^-,\ \mathsf{Cathode},\ \mathsf{Electrolyte}\}.$$

Elements of $\mathcal{P}^2(V_0)$ are collections of nonempty subsets of V_0. Define three 2–supervertices

$$A_1 = \big\{\{\mathrm{Fe}^{3+}, e^-\},\ \{\mathrm{Fe}^{2+}\}\big\},$$

$$A_2 = \big\{\{\mathsf{Cathode}\},\ \{e^-\}\big\},$$

$$A_3 = \big\{\{\mathsf{Electrolyte}\},\ \{\mathrm{Fe}^{3+}\},\ \{\mathrm{Fe}^{2+}\}\big\}.$$

Set $V = \{A_1, A_2, A_3\} \subseteq \mathcal{P}^2(V_0)$ and

$$E = \big\{\{A_1, A_2\},\ \{A_2, A_3\}\big\} \subseteq \mathcal{P}(V) \setminus \{\emptyset\}.$$

Then $\mathrm{SHG}^{(2)} = (V, E)$ is a 2–SuperHyperGraph encoding a half–reaction group (A_1), an electrode–interface group (A_2), and an environment group (A_3), with superedges capturing their coupling.

Example 18 (Electrobiochemical SuperHyperGraph: Enzyme–Coupled ET)**.** Let

$$V_0 = \{\mathrm{NAD}^+,\ \mathrm{NADH},\ \mathrm{H}^+,\ e^-,\ \mathsf{Anode},\ \mathsf{FDH}\}.$$

Define

$$A_1 = \big\{\{\mathrm{NAD}^+\},\ \{\mathrm{NADH}\}\big\},$$

$$A_2 = \big\{\{\mathrm{NADH}\},\ \{\mathrm{NAD}^+, \mathrm{H}^+, e^-, \mathsf{Anode}\}\big\},$$

$$A_3 = \big\{\{\mathsf{FDH}\},\ \{\mathrm{NAD}^+, \mathrm{NADH}\}\big\}.$$

Let $V = \{A_1, A_2, A_3\}$ and $E = \big\{\{A_1, A_2\}, \{A_2, A_3\}\big\}$. Then $\mathrm{SHG}^{(2)} = (V, E)$ captures a bioreaction redox pair (A_1), an electrode ET grouping (A_2), and enzyme coupling (A_3), with superedges representing pathway connectivity.

2.4 | Physicochemical Graph

Physicochemistry explores physical principles underlying chemical systems, emphasizing thermodynamics, kinetics, quantum mechanics, and molecular interactions in diverse environments [35, 36, 37, 38]. A *Physicochemical Graph* encodes chemical species and physical processes, such as diffusion, heat transfer, and reactions, linking molecular interactions with physical dynamics.

Definition 21 (Physicochemical Graph)**.** A *physicochemical graph* augments chemical reaction structure with physical transport/field processes:

$$\mathcal{G}_{\mathrm{pc}} = (S, P, E_{\mathrm{st}}, E_{\mathrm{phys}}, \mathrm{src}, \mathrm{tgt}, \mathrm{typ}_S, \mathrm{typ}_P, \nu, \mathrm{region}, \kappa),$$

where:

- S are species; P includes Reaction and Physical processes (diffusion, convection, heat exchange).

- E_{st} are stoichiometric edges; $E_{\mathrm{phys}} \subseteq (S \times P) \cup (P \times S)$ are *transport/field* edges.
- region : $S \to \mathcal{R}$ assigns spatial regions (compartments or mesh cells).
- $\kappa : E_{\mathrm{phys}} \to \mathbb{R}_{\geq 0}$ encodes physical coefficients (e.g. diffusion, permeability, heat conductance).

Reactions satisfy elemental conservation as in the biochemical graph; physical processes satisfy constitutive symmetry/positivity through κ (e.g. nonnegative diffusivity), and edges in E_{phys} connect species across regions according to the chosen discretization.

Example 19 (Physicochemical Graph: Diffusion and Reaction in Two Regions)**.** Chemical species $S = \{A, B, C\}$ and regions $\mathcal{R} = \{\Omega_1, \Omega_2\}$. Reaction node p_{rxn} (in Ω_2):

$$A@\Omega_2 \xrightarrow[\nu=1]{} p_{\mathrm{rxn}},$$
$$B@\Omega_2 \xrightarrow[\nu=1]{} p_{\mathrm{rxn}},$$
$$p_{\mathrm{rxn}} \xrightarrow[\nu=1]{} C@\Omega_2.$$

Transport node p_{tr} (diffusion of A from Ω_1 to Ω_2) with coefficient $\kappa = D_A$:

$$A@\Omega_1 \xrightarrow[\nu=1]{D_A} p_{\mathrm{tr}}, \qquad p_{\mathrm{tr}} \xrightarrow[\nu=1]{D_A} A@\Omega_2.$$

Species-wise conservation holds on p_{tr} (one A leaves Ω_1 and enters Ω_2). The physicochemical graph couples transport with the local reaction $A + B \to C$ in Ω_2.

Definition 22 (Physicochemical Hypergraph)**.** Let S be chemical species, $\mathcal{R}$ a finite set of spatial regions, and consider the lifted set $S@\mathcal{R} = \{(s, r) : s \in S,\, r \in \mathcal{R}\}$. A *physicochemical hypergraph* is

$$\mathcal{H}_{\mathrm{pc}} = \big(S@\mathcal{R},\ \mathcal{E}_{\mathrm{rxn}},\ \mathcal{E}_{\mathrm{tr}},\ \alpha, \beta, \tau, \kappa, w\big),$$

with:

- $\mathcal{E}_{\mathrm{rxn}}$ reaction hyperedges, each h having $\alpha_h, \beta_h : S@\mathcal{R} \to \mathbb{N}_0$ (local reactions);
- $\mathcal{E}_{\mathrm{tr}}$ *transport* hyperedges; each $u \in \mathcal{E}_{\mathrm{tr}}$ has tail/head multisets α_u, β_u such that $\sum_r \alpha_u(s, r) = \sum_r \beta_u(s, r)$ for every s (pure redistribution);
- $\tau : \mathcal{E}_{\mathrm{tr}} \to \{\mathsf{Diffusion}, \mathsf{Convection}, \mathsf{Heat}\}$ types transport; $\kappa : \mathcal{E}_{\mathrm{tr}} \to \mathbb{R}_{\geq 0}$ stores physical coefficients (e.g., diffusivity/permeability);
- w assigns optional weights to all hyperedges.

Each $h \in \mathcal{E}_{\mathrm{rxn}}$ obeys elemental conservation $\sum_{x \in S@\mathcal{R}} (\beta_h(x) - \alpha_h(x))\, c(x) = 0$ for any conserved c, while each $u \in \mathcal{E}_{\mathrm{tr}}$ redistributes mass/energy across regions without net creation.

Example 20 (Physicochemical HyperGraph: Transport–Reaction Coupling)**.** Let spatially tagged species be

$$V = \{A_{\Omega_1},\ A_{\Omega_2},\ B_{\Omega_2},\ C_{\Omega_2},\ \mathsf{Diff},\ \mathsf{Rxn}\}.$$

Define two hyperedges:

$$h_{\mathrm{diff}} = (\mathsf{Transport},\ \{A_{\Omega_1}, A_{\Omega_2}, \mathsf{Diff}\}),$$
$$h_{\mathrm{rxn}} = (\mathsf{Reaction},\ \{A_{\Omega_2}, B_{\Omega_2}, C_{\Omega_2}, \mathsf{Rxn}\}).$$

Assign

$$\nu_{\mathrm{diff}}(A_{\Omega_1}) = -1,\ \nu_{\mathrm{diff}}(A_{\Omega_2}) = +1,\ \nu_{\mathrm{diff}}(\mathsf{Diff}) = 0,$$

and

$$\nu_{\mathrm{rxn}}(A_{\Omega_2}) = -1,\ \nu_{\mathrm{rxn}}(B_{\Omega_2}) = -1,\ \nu_{\mathrm{rxn}}(C_{\Omega_2}) = +1,\ \nu_{\mathrm{rxn}}(\mathsf{Rxn}) = 0,$$

representing diffusion of A from Ω_1 to Ω_2 and a local reaction $A + B \to C$ in Ω_2.

Definition 23 (Physicochemical SuperHyperGraph)**.** Let S_0 be chemical species and $\mathcal{R}$ a finite set of spatial regions. Set $X_0 := S_0 \times \mathcal{R}$ and fix n with $V \subseteq \mathcal{P}^n(X_0)$ finite. A *physicochemical superhypergraph* is

$$\mathcal{H}_{\mathrm{pc}}^{(n)} = (X_0, V, \mathcal{E}_{\mathrm{rxn}}, \mathcal{E}_{\mathrm{tr}}, \alpha, \beta, \tau, \kappa, w),$$

where:

- $\mathcal{E}_{\mathrm{rxn}}$ are *reaction* superhyperedges with superstoichiometries $\alpha_h, \beta_h : V \to \mathbb{N}_0$;
- $\mathcal{E}_{\mathrm{tr}}$ are *transport* superhyperedges u with superstoichiometries α_u, β_u;
- $\tau : \mathcal{E}_{\mathrm{tr}} \to \{\mathsf{Diffusion}, \mathsf{Convection}, \mathsf{Heat}\}$ is a transport type;
- $\kappa : \mathcal{E}_{\mathrm{tr}} \to \mathbb{R}_{\geq 0}$ encodes physical coefficients; w are optional weights.

Let $\widehat{\alpha}, \widehat{\beta}$ be the induced stoichiometries on X_0 via flattening. Then for each reaction h and conserved composition $c : S_0 \to \mathbb{Z}^d$,

$$\sum_{(s,r)\in X_0} \big(\widehat{\beta}_h(s,r) - \widehat{\alpha}_h(s,r)\big)\, c(s) = 0,$$

and for each transport u and each $s \in S_0$,

$$\sum_{r\in\mathcal{R}} \big(\widehat{\beta}_u(s,r) - \widehat{\alpha}_u(s,r)\big) = 0 \quad \text{(species-wise conservation across regions).}$$

Example 21 (Physicochemical SuperHyperGraph: Transport–Reaction Pipeline)**.** Let

$$V_0 = \{A@\Omega_1,\ A@\Omega_2,\ B@\Omega_2,\ C@\Omega_2,\ \mathsf{Diff},\ \mathsf{Rxn}\}.$$

Define stage groups

$$A_1 = \big\{\{A@\Omega_1\},\ \{A@\Omega_2, \mathsf{Diff}\}\big\},$$
$$A_2 = \big\{\{A@\Omega_2, B@\Omega_2\},\ \{C@\Omega_2, \mathsf{Rxn}\}\big\}.$$

Set $V = \{A_1, A_2\}$ and $E = \big\{\{A_1, A_2\}\big\}$. Then $\mathrm{SHG}^{(2)} = (V, E)$ is a 2–SuperHyperGraph chaining a diffusion stage (A_1) to a local reaction stage (A_2).

2.5 | physicobiochemical Graph

Physicobiochemistry integrates physical principles with biochemistry, examining molecular forces, structural dynamics, and energy transformations in biological macromolecules and cellular systems [39, 40, 41]. A *Physicobiochemical Graph* unifies biochemical pathways with physical processes, modeling how transport, diffusion, and regulation together influence cellular metabolic networks.

Definition 24 (Physicobiochemical Graph)**.** A *physicobiochemical graph* combines biochemical reactions, regulation, and physical transport:

$$\mathcal{G}_{\mathrm{pbc}} = (S, P, E_{\mathrm{st}}, E_{\mathrm{reg}}, E_{\mathrm{phys}},$$
$$\mathrm{src}, \mathrm{tgt}, \mathrm{typ}_S, \mathrm{typ}_P, \nu, \mathrm{comp}, \mathrm{region}, \kappa).$$

Here S includes metabolites and macromolecules; P contains Reaction, Regulation, and Physical nodes. Edges E_{st} (stoichiometric), E_{reg} (regulatory), and E_{phys} (transport/fields) are as above. Conservation constraints apply at Reaction nodes, while κ constrains physical edges. This structure supports multi-scale models where transport limits and regulatory logic jointly shape pathway fluxes.

Example 22 (Physicobiochemical Graph: Glucose Uptake, Hexokinase, and Feedback)**.** Species

$$S = \{\mathrm{Glc}, \mathrm{ATP}, \mathrm{G6P}, \mathrm{ADP}\}$$

and regions

$$\mathcal{R} = \{\Omega_{\mathrm{blood}}, \Omega_{\mathrm{cyto}}\}$$

. Transport node p_{glc} (permeation) with coefficient $\kappa = P_{\mathrm{GLUT}}$:

$$\mathrm{Glc}@\Omega_{\mathrm{blood}} \xrightarrow[\nu=1]{P_{\mathrm{GLUT}}} p_{\mathrm{glc}},$$

$$p_{\mathrm{glc}} \xrightarrow[\nu=1]{P_{\mathrm{GLUT}}} \mathrm{Glc}@\Omega_{\mathrm{cyto}}.$$

Reaction node p_{HK} (hexokinase in cytosol):

$$\mathrm{Glc}@\Omega_{\mathrm{cyto}} \xrightarrow[\nu=1]{} p_{\mathrm{HK}},$$

$$\mathrm{ATP@}\Omega_{\mathrm{cyto}} \xrightarrow[\nu=1]{} p_{\mathrm{HK}}, \;\; p_{\mathrm{HK}} \xrightarrow[\nu=1]{} \mathrm{G6P@}\Omega_{\mathrm{cyto}},$$
$$p_{\mathrm{HK}} \xrightarrow[\nu=1]{} \mathrm{ADP@}\Omega_{\mathrm{cyto}}.$$

Regulatory edge (feedback inhibition): $\mathrm{G6P@}\Omega_{\mathrm{cyto}} \dashv p_{\mathrm{HK}}$. This physicobiochemical graph integrates membrane transport, metabolism, and regulation.

Definition 25 (Physicobiochemical Hypergraph)**.** A *physicobiochemical hypergraph* combines biochemical reaction/regulation with spatial transport:

$$\mathcal{H}_{\mathrm{pbc}} = \big(S@\mathcal{R},\ \mathcal{E}_{\mathrm{rxn}},\ \mathcal{E}_{\mathrm{reg}},\ \mathcal{E}_{\mathrm{tr}}, \alpha, \beta, R^+, R^-, \tau, \kappa, w\big),$$

where $S@\mathcal{R}$, $\mathcal{E}_{\mathrm{rxn}}$, $\mathcal{E}_{\mathrm{tr}}$, and (τ, κ) are as in the physicochemical case, and $\mathcal{E}_{\mathrm{reg}}$ provides regulatory hyperedges (activators/inhibitors) that constrain the feasibility or rate of reaction hyperedges. Reactions satisfy elemental conservation; transport hyperedges conserve per-species totals across regions.

Example 23 (Physicobiochemical HyperGraph: GLUT Transport and Hexokinase with Feedback)**.** Vertices

$$V = \{\mathrm{Glc}_{\mathrm{blood}},\ \mathrm{Glc}_{\mathrm{cyto}},\ \mathrm{ATP}_{\mathrm{cyto}},$$
$$\mathrm{ADP}_{\mathrm{cyto}},\ \mathrm{G6P}_{\mathrm{cyto}},\ \mathsf{GLUT},\ \mathsf{HK},\ \mathsf{Reg}\}.$$

Hyperedges:

$$h_{\mathrm{tr}} = (\mathsf{Transport},\ \{\mathrm{Glc}_{\mathrm{blood}},\ \mathrm{Glc}_{\mathrm{cyto}},\ \mathsf{GLUT}\}),$$
$$h_{\mathrm{hk}} = (\mathsf{Reaction},\ \{\mathrm{Glc}_{\mathrm{cyto}},\ \mathrm{ATP}_{\mathrm{cyto}},\ \mathrm{G6P}_{\mathrm{cyto}},\ \mathrm{ADP}_{\mathrm{cyto}},\ \mathsf{HK}\}),$$
$$h_{\mathrm{fb}} = (\mathsf{Regulation},\ \{\mathrm{G6P}_{\mathrm{cyto}},\ \mathsf{HK},\ \mathsf{Reg}\}).$$

Stoichiometries:

$$\nu_{\mathrm{tr}}(\mathrm{Glc}_{\mathrm{blood}}) = -1,\ \ \nu_{\mathrm{tr}}(\mathrm{Glc}_{\mathrm{cyto}}) = +1,$$
$$\nu_{\mathrm{tr}}(\mathsf{GLUT}) = 0,$$
$$\nu_{\mathrm{hk}}(\mathrm{Glc}_{\mathrm{cyto}}) = -1,\ \ \nu_{\mathrm{hk}}(\mathrm{ATP}_{\mathrm{cyto}}) = -1,$$
$$\nu_{\mathrm{hk}}(\mathrm{G6P}_{\mathrm{cyto}}) = +1,\ \ \nu_{\mathrm{hk}}(\mathrm{ADP}_{\mathrm{cyto}}) = +1,$$
$$\nu_{\mathrm{hk}}(\mathsf{HK}) = 0,$$

while h_{fb} encodes inhibition (no stoichiometry) of HK by $\mathrm{G6P}_{\mathrm{cyto}}$.

Definition 26 (Physicobiochemical SuperHyperGraph)**.** With $X_0 = S_0 \times \mathcal{R}$ and $V \subseteq \mathcal{P}^n(X_0)$ finite, a *physicobiochemical superhypergraph* is

$$\mathcal{H}^{(n)}_{\mathrm{pbc}} = (X_0, V, \mathcal{E}_{\mathrm{rxn}}, \mathcal{E}_{\mathrm{reg}}, \mathcal{E}_{\mathrm{tr}}, \alpha, \beta, R^+, R^-, \tau, \kappa, w),$$

combining reaction/regulatory superhyperedges with transport superhyperedges. Induced balances satisfy elemental conservation on reactions (as in the physicochemical case) and species-wise conservation on transport edges; regulatory superhyperedges constrain reaction feasibility or rates but do not carry stoichiometry.

Example 24 (Physicobiochemical SuperHyperGraph: GLUT Transport, HK Reaction, Feedback)**.** Let

$$V_0 = \{\mathrm{Glc@blood},\ \mathrm{Glc@cyto},\ \mathrm{ATP@cyto},\ \mathrm{ADP@cyto},\ \mathrm{G6P@cyto},\ \mathsf{GLUT},\ \mathsf{HK},\ \mathsf{Reg}\}.$$

Define

$$A_1 = \big\{\{\mathrm{Glc@blood}\},\ \{\mathrm{Glc@cyto}, \mathsf{GLUT}\}\big\},$$
$$A_2 = \big\{\{\mathrm{Glc@cyto}, \mathrm{ATP@cyto}\},\ \{\mathrm{G6P@cyto}, \mathrm{ADP@cyto}, \mathsf{HK}\}\big\},$$
$$A_3 = \big\{\{\mathrm{G6P@cyto}\},\ \{\mathsf{HK}, \mathsf{Reg}\}\big\}.$$

With $V = \{A_1, A_2, A_3\}$ and

$$E = \big\{\{A_1, A_2\}, \{A_2, A_3\}\big\},$$

the 2–SuperHyperGraph links transport (A_1), metabolism (A_2), and feedback control (A_3).

2.6 | Medichemical Graph

In this paper, we use the term *medichemistry* to refer to medical chemistry [42, 43, 44, 45]. Medichemistry focuses on the design, synthesis, and analysis of therapeutic compounds, applying chemical principles to elucidate pharmacological activity, mechanisms of action, and clinical applications. A *Medichemical Graph* represents drugs, molecular targets, biological pathways, and diseases as typed vertices, with labeled directed edges encoding therapeutic, inhibitory, metabolic, and other clinically relevant interactions.

Definition 27 (Medichemical Graph)**.** A *medichemical graph* is a typed, directed, labeled multigraph

$$\mathcal{G}_{\text{medchem}} = (V, E, \text{src}, \text{tgt}, \text{typ}_V, \text{typ}_E, \lambda_V, \lambda_E, w),$$

with:

- V a finite set of entities with typing $\text{typ}_V : V \to \{\textsf{Drug}, \textsf{Target}, \textsf{Pathway}, \textsf{Disease}, \textsf{AdverseEvent}, \textsf{Enzyme}\}$.
- $E \subseteq V \times V$ directed edges with typing

$$\text{typ}_E : E \to \{\textsf{binds}, \textsf{inhibits}, \textsf{activates}, \textsf{treats}, \textsf{contraindicated\_with}, \textsf{metabolized\_by}\}$$

  .
- λ_V and λ_E are (optional) attribute maps (e.g. affinities, IC_{50}, indications, evidence codes).
- $w : E \to \mathbb{R}_{\geq 0}$ encodes confidence/strength (e.g. evidence-weight or binding score).

Direction and edge type specify role semantics (e.g. (d, t) with type $\textsf{binds}$ means drug d binds target t; (d, z) with type $\textsf{treats}$ means drug d treats disease z). This graph supports downstream tasks such as indication expansion, off-target risk inference, and mechanism-of-action reasoning.

Example 25 (Medichemical Graph: Warfarin Mechanistic Relations)**.** Entities

$$V = \{\textsf{Warfarin} : \textsf{Drug},\ \textsf{VKORC1} : \textsf{Target},\ \textsf{Thromboembolism} : \textsf{Disease},$$

$$\textsf{CYP2C9} : \textsf{Enzyme},\ \textsf{Bleeding} : \textsf{AdverseEvent}\}.$$

Directed typed edges (examples):

$$(\textsf{Warfarin} \xrightarrow{\textsf{binds}} \textsf{VKORC1}),$$

$$(\textsf{Warfarin} \xrightarrow{\textsf{inhibits}} \textsf{VKORC1}),$$

$$(\textsf{Warfarin} \xrightarrow{\textsf{treats}} \textsf{Thromboembolism}),$$

$$(\textsf{Warfarin} \xrightarrow{\textsf{metabolized\_by}} \textsf{CYP2C9}),$$

$$(\textsf{Warfarin} \xrightarrow{\textsf{contraindicated\_with}} \textsf{Bleeding}).$$

This medichemical graph supports reasoning about mechanism of action, therapy, metabolism, and safety signals.

Definition 28 (Medichemical Hypergraph)**.** Let V be a finite set of entities with typing $\text{typ}_V : V \to \mathcal{C}$ where $\mathcal{C} = \{\textsf{Drug}, \textsf{Target}, \textsf{Pathway}, \textsf{Disease}, \textsf{AdverseEvent}, \textsf{Enzyme}, \textsf{Variant}\}$. Let $\mathcal{R}$ be a finite set of relation types equipped with arity/signature

$$\text{ar} : \mathcal{R} \to \mathbb{N}_{\geq 1}, \qquad \text{sig} : \mathcal{R} \to \bigcup_{k \geq 1} \mathcal{C}^k.$$

A *medichemical hypergraph* is

$$\mathcal{H}_{\text{medchem}} = \big(V,\ \mathcal{E},\ \text{ar},\ \text{sig},\ w\big),$$

where each hyperedge $e \in \mathcal{E}$ has the form $e = (r, (v_1, \ldots, v_k))$ with $r \in \mathcal{R}$, $k = \text{ar}(r)$, $v_i \in V$, and the typing constraint $\text{typ}_V(v_i) = \text{sig}(r)_i$ for all $i = 1, \ldots, k$. The optional weight $w(e) \in \mathbb{R}_{\geq 0}$ can encode confidence, binding strength, or evidence. This formulation captures multi-entity relations (e.g., $\textsf{drug–target–variant–disease}$ tuples) in a single typed hyperedge.

Example 26 (Medichemical HyperGraph: Drug–Target–Disease–Enzyme Relations)**.** Let

$$V = \{\mathsf{Warfarin},\ \mathsf{VKORC1},\ \mathsf{Thromboembolism},\ \mathsf{CYP2C9},\ \mathsf{Bleeding}\}.$$

Introduce typed hyperedges (multi-entity medical relations):

$$h_1 = (\mathsf{binds/inhibits},\ \{\mathsf{Warfarin},\ \mathsf{VKORC1}\}),\quad h_2 = (\mathsf{treats},\ \{\mathsf{Warfarin},\ \mathsf{Thromboembolism}\}),$$

$$h_3 = (\mathsf{metabolized\_by},\ \{\mathsf{Warfarin},\ \mathsf{CYP2C9}\}),\quad h_4 = (\mathsf{contraindicated\_with},\ \{\mathsf{Warfarin},\ \mathsf{Bleeding}\}).$$

The set $E = \{h_1, h_2, h_3, h_4\}$ forms a medichemical hypergraph capturing mechanism of action, therapeutic use, metabolism, and safety constraint in a unified multiway structure.

Definition 29 (Medichemical SuperHyperGraph)**.** Let E_0 be a finite set of biomedical/chemical entities with typing $\mathrm{typ} : E_0 \to \mathcal{C}$, where $\mathcal{C} = \{\mathsf{Drug}, \mathsf{Target}, \mathsf{Pathway}, \mathsf{Disease}, \mathsf{AdverseEvent}, \mathsf{Enzyme}, \mathsf{Variant}\}$. Fix n and choose a finite set of supervertices $V \subseteq \mathcal{P}^n(E_0)$. Let $\mathcal{R}$ be a finite set of relation types with

$$\mathrm{ar} : \mathcal{R} \to \mathbb{N}_{\geq 1}, \qquad \mathrm{sig} : \mathcal{R} \to \bigcup_{k \geq 1} \mathcal{C}^k.$$

A *medichemical superhypergraph* is

$$\mathcal{H}^{(n)}_{\mathrm{medchem}} = (E_0, V, \mathcal{E}, \mathrm{ar}, \mathrm{sig}, w),$$

where a superhyperedge is a typed tuple

$$e = (r, (A_1, \ldots, A_k)) \quad \text{with} \quad r \in \mathcal{R},\ k = \mathrm{ar}(r),\ A_i \in V,$$

subject to the *admissibility constraint*

$$\forall i \in \{1, \ldots, k\}\ \forall e' \in \mathrm{Flat}_n(A_i) \colon\ \mathrm{typ}(e') = \mathrm{sig}(r)_i.$$

The optional weight $w(e) \in \mathbb{R}_{\geq 0}$ encodes confidence/strength (e.g. binding evidence, clinical association). This generalizes typed knowledge hyperedges to hierarchical (supervertex) arguments.

Example 27 (Medichemical SuperHyperGraph: Mechanism–Therapy–Metabolism–Safety)**.** Let

$$V_0 = \{\mathsf{Warfarin},\ \mathsf{VKORC1},\ \mathsf{Thromboembolism},\ \mathsf{CYP2C9},\ \mathsf{Bleeding}\}.$$

Define the following 2–supervertices:

$$A_1 = \big\{\{\mathsf{Warfarin}, \mathsf{VKORC1}\}\big\},\quad A_2 = \big\{\{\mathsf{Warfarin}, \mathsf{Thromboembolism}\}\big\},$$

$$A_3 = \big\{\{\mathsf{Warfarin}, \mathsf{CYP2C9}\}\big\},\quad A_4 = \big\{\{\mathsf{Warfarin}, \mathsf{Bleeding}\}\big\}.$$

Set $V = \{A_1, A_2, A_3, A_4\}$ and $E = \big\{\{A_1, A_2, A_3\},\ \{A_1, A_4\}\big\}$. Then $\mathrm{SHG}^{(2)} = (V, E)$ forms a 2–SuperHyperGraph where A_1 (mechanism) connects jointly to therapy and metabolism (A_2, A_3) and separately to a safety constraint (A_4).

3 | Conclusion

In this paper, we investigated whether new concepts such as the *Biochemical Graph*, *Electrochemical Graph*, *Physicochemical Graph*, and *Medichemical Graph* can be formally defined. Future research may explore extensions employing *Fuzzy Graphs* [46, 47, 48], *Intuitionistic Fuzzy Graphs* [49, 50], *Neutrosophic Graphs* [51, 52, 53], and *Plithogenic Graphs* [54, 55, 56], providing enriched frameworks for uncertainty modeling and domain-specific applications.

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Use of Generative AI and AI-Assisted Tools

We use generative AI and AI-assisted tools for tasks such as English grammar checking, and We do not employ them in any way that violates ethical standards.

Data Availability

This manuscript presents purely conceptual work without empirical data. Scholars interested in these ideas are invited to undertake experimental or case-study research to substantiate and extend the proposed frameworks.

Ethical Approval

This paper involves no human or animal subjects and thus did not require ethics committee review or approval.

Conflicts of Interest

The authors declare that there are no competing interests concerning the content or publication of this article.

Disclaimer

The theoretical models and propositions herein have not yet been subjected to practical validation. Readers should independently verify all citations and be aware that inadvertent inaccuracies may remain. The opinions expressed are those of the authors and do not necessarily represent the views of affiliated organizations.

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